THEORY OF THE SATOH PROBE IN ELECTROGASDYNAMICS

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The theory of an electric probe for studies of the outer zone of a constant current corona discharge, which may be regarded as an electrogasdynamic (EGD) flow with zero gas flow velocity, was propounded in [1]. The theory of probe measurements on AC coronas was further developed and extended in [2, 3].

A simplified theory of the probe was introduced in [4]. In this theory the potential and volume charge density at the point involved are determined from the measured values of the floating potential and the current to a grounded probe. (The floating potential of a probe is the potential at which the probe current is zero.) This method does not require plotting of the probe characteristics and may be used for weakly-charged EGD flows.

The problem of the optimal form of the probe when the floating potential equals the potential at the point in question was considered in [5].

The results of an experimental study using a Satoh probe of the electric field in a corona discharge in the presence of a gas flow which impedes charged particle motion were presented in [6]. However, the nature of the probe characteristics when the flow velocity vectors are not colinear with the electric field intensity was not discussed.

In the present paper we consider the properties of the volt-ampere characteristics of a cylindrical probe located in plane-compressible and incompressible EGD flows with relatively large space charge densities \( q > 10^{-4} \text{ Coul/m}^3 \) and arbitrary directions of the flow velocity and electric field vectors. It is shown that using a Satoh probe it is possible to determine the absolute value of the electric field intensity only when the angle between these vectors is less than or equal to \( \pm \pi/2 \).

1. Volt-Ampere Characteristic of a Cylindrical Probe in an Incompressible Flow. The theory of the Satoh probe in a corona discharge is based on the following assumptions [2]: a) diffusion currents are negligibly small; b) the electric field being studied, \( E_0 \), is uniform in the neighborhood of the probe over distances comparable to its characteristic size; c) the perturbation of the field due to changes in the space charge density at and in the neighborhood of the probe location is negligibly small for \( I_p \neq 0 \); d) the condition \( I_p/\lambda \ll 1 \), holds, where \( I_p \) is the probe current (per unit length in the case of a cylindrical probe) and \( \lambda \) is the current in the discharge gap. These assumptions allow us to regard the space charge density in the neighborhood of the cylindrical probe as practically constant and equal to the charge density, \( q_0 \), at the given point when the probe is not present [2].

In the following we shall regard these assumptions as satisfied even in the case of an EGD flow.

We shall consider a cylindrical probe of radius \( r_0 \) located in a two-dimensional ideal incompressible EGD flow with arbitrary directions of the gas velocity \( V_0 \) and electric field intensity \( E_0 \) vectors. We shall assume that the resulting velocity of the charged particles, \( V_0 + kE_0 \), is constant over distances much larger than the characteristic dimension of the probe (k is the ion mobility) [2, 4].

The complex potential of the flow near a probe with charge \( \tau \) per unit length has the form

\[
W = \psi + i\Phi = -i(V_0 + kE_0)e^{-\alpha z} - i(V_0 - kE_0)e^{\alpha r_0^2 / z - iB \ln z} \quad (1.1)
\]


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Here \( \Phi \) and \( \psi \) are the potential and stream function of the EGD flow; \( z = x + iy \); \( \alpha \) is the angle between the real axis, \( x \), and the vector \( \mathbf{E}_0 \) (the velocity \( \mathbf{V}_0 \) is directed along the \( x \) axis); \( B = k\tau(2\pi\varepsilon)^{-1} \), where \( \varepsilon \) is the dielectric constant; and \( \tau \) depends on the probe potential and may be controlled using an auxiliary supply.

Using Eq. (1), we evaluate the stream function and the radial component of the resulting ion velocity, \( V_T' = V_T/V_0 \), at the probe surface:

\[
\frac{\psi}{V_T} = (r'' - 1/r') \sin \theta + R \left[ \frac{2m}{r'' + 1/r'} \sin(\theta - \alpha) + 2m \right] \quad (1.2)
\]

\[
V_{T'} = 2R \left[ \cos(\theta - \alpha) + m \right], \quad m = 0.5B/k\tau|\mathbf{E}_q|, \quad (1.3)
\]

It follows from Eq. (1.3) that for \( m \leq 1 \), \( V_{T'} \mid_{r'' = 1} < 0 \) and the probe collects ions over its entire surface. The total current to the probe is determined by (the linear part of the probe characteristic)

\[
I_p = -2V_0 \int_{0}^{\pi/2} V_{T'} \mid_{r'' = 1} d\theta = -k\tau q e^{-i} \quad (\tau \leq 4\pi\tau_0|\mathbf{E}_q|) \quad (1.4)
\]

If \( \tau = C(U - U_0) \) (\( C \) is the capacitance of the probe-electrode system; \( U \), the probe electric potential; and \( U_0 \) the potential at the point) [2], then

\[
I_s = kqC(U_0 - U) e^{-i} \quad (1.5)
\]

Equation (1.5) is correct for an arbitrary angle \( \alpha \) and permits determination of the potential at the observed point from the experimental volt-ampere characteristic of the probe (from the point of intersection of the linear part of the characteristic with the potential axis in Fig. 1), as well as the conductivity \( kq \) at the probe position or the space charge density, if the mobility of the particles is known. We note that Eqs. (1.4) and (1.5) do not depend on \( R \) and agree with the corresponding expressions for the linear part of the probe characteristic in a corona discharge [2].

If the hydrodynamic perturbations of the EGD flow and the perturbations of the electric field due to the polarization of the probe are negligibly small compared with the perturbations due to its charge \( \tau \) (this is so for \( |\tau| >> 2\pi\tau_0V_0k^{-1}|1 - R_e| \)), then a cylindrical probe may be regarded as a linear drain (\( \tau \rightarrow 0 \)). In this case, instead of Eq. (1.1) we may write

\[
W^* = (-i/2\pi)(2\pi L \sigma - I_0 \ln z) \quad (1.6)
\]

in the neighborhood of the probe since we assumed the volume charge density to be constant. Here \( W^* \) is the complex potential of the electric current density, \( \mathbf{j} = \mathbf{V}_0 q_0 \), and \( j_0 \) is the current density in the unperturbed flow.

Using Eq. (1.6) we can determine the coordinate of the critical point, \( x_0 = I_0/2\pi j_0 \), and the maximum halfwidth of the ion attraction zone \( h = I_0/2\pi j_0 \), which is bounded by the branched stream lines given by \( y = x \tan(y/h) \). Taking \( y/h = 0.99 \), we find the characteristic radius of the perturbation zone of the current density to be \( R = 31.9h \approx 16I_0/j_0 \) within which \( j_0 \leq 0.99j_0 \). For measurements with a cylindrical probe in an inhomogeneous EGD flow it is necessary that the current density \( j_0(x, y) \) remain practically constant in a circle of radius \( R \) γ ≥ R.

If \( |\tau| < 2\pi\tau_0V_0k^{-1}|1 - R_e| \), then the perturbed charged particle velocity is due mainly to polarization effects and hydrodynamic flow around the cylinder. In this case, as follows from Eq. (1.2), the perturbations propagate over a distance \( \sim 10r_0 |1 - R_e| \), which for \( q_0 = \text{const} \) defines the characteristic dimension of the zone in which the current density \( j_0(x, y) \) remain practically constant in a circle of radius \( R_1 \geq R \).

It can be shown that Eq. (1.5) describes the linear part of the probe characteristic for a probe of arbitrary shape. For a spherical probe, assuming \( U = 0 \), \( U_0 = U_s \), and \( C = 4\pi\tau_0 \) (\( U_s \) is the floating potential; \( C \), the capacitance of an isolated sphere), we find a relation between the current to the ground probe and the floating potential: