Correspondence Principle for the Quantum Net

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Finkelstein’s suggestion for a flexible logic is taken up in the context of his causal net theory. We interpret on the net certain concepts that are first expressed in terms of the canonical “flexible logic” of the macroscopic world, namely, the logic of sheaves over the manifold model, here taken to be flat. From this we infer a correspondence principle in the form of a simple (model-dependent) semantics which translates certain concepts between the purely quantum world of the net and the familiar classical-quantum hybridized world of the macroscopic model. As an application, we derive and solve the reticular version of the massless Dirac equation by analyzing the Dirac operator on the net, where its behavior is easily apprehended.

1. INTRODUCTION

A continuing series of works laid the foundations for a theory which is antecedent to both relativity and quantum theory and thereby achieves their fusion at a deep level (see in particular Finkelstein, 1969a,b, 1972, 1974, 1987, 1988, 1989, and Finkelstein et al., 1974.) A fundamental step in the construction of this theory is the extension and generalization of the von Neumann interpretation of quantum mechanics in terms of a logic of (quantum) propositions, which is then employed to produce a quantal description of those entities which may be presumed to appear macroscopically as spacetime points. The power of this idea is immediately apparent when it is combined with a causality requirement: for then the major kinematical features of relativity emerge spontaneously—this is achieved already in the first paper (Finkelstein, 1969a). In subsequent work (Finkelstein, 1987, 1990; Finkelstein and Hallidy, 1990), shortcomings of the original von Neumann-like quantum logics were isolated and repaired, a process culminating in the unveiling of a beautifully symmetric object, the extensor algebra (Barnabei

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et al., 1985), which, when interpreted as describing a theory of "quantum" sets, shares many of the properties of ordinary set theory. The substratum of the world is then postulated to be a plexus of primitive spinor like "events" which can combine only according to the laws of quantum extensor logic together with a causal structure, which is also necessarily expressed in terms of the quantum logic. The challenge is to show how ordinary spacetime, with some version of quantum mechanics attached, emerges from this plexus as some kind of limit. A mechanism to account for the initial phase is convincingly argued in Finkelstein (1988, 1989). In this scenario, very roughly speaking, Cooper-like spinor–conjugate spinor pairs form as cooling takes place in the chaotic (but causal) primordial event-vacuum, and a transition is made to a new vacuum, the net. These bosonlike pairs transform as Lorentz vectors and, when aggregated in large numbers, form into vectors in a certain Minkowski space, which apparently corresponds to a local structure on the classical limit manifold, such as the tangent space, or null-cone, at a point. The implementation of this scheme, which involves an intermediate Fock-like algebra, gives a convincing account also of the origin of such basic quantum mechanical items as the Heisenberg communication relations.

The very success of these procedures in giving a good account of the local structure of classical spacetime (plus quantum mechanics) serves to underline the problem of finding a general principle of correspondence mediating between the rigid quantal world of the net and the more familiar (albeit chimerical) world of curvaceous classical spacetime (with quantum fields attached). One source of difficulty may be ascribed to the rigidity of the underlying q-set theory, or logic, itself. Finkelstein has argued the case for a "warped" or flexible logic at this deep level (Finkelstein, 1969b, Section III): "If a flexible logic is possible at all, it may be rich enough to account for much more of the phenomena we see at the higher levels than we usually regard as logical in origin."

This remark foreshadows the most important development in modern pure ("classical") logic, which occurred in the year of its publication, namely, the invention by Lawvere and Tierney of topos theory. The motivation for this development lay within pure logic itself, where the notions of classical set theory had been found too rigid. For example, some models of ZF were found by P. J. Cohen in 1963 to falsify certain propositions (the axiom of choice, the continuum hypothesis), while other models had been shown by Gödel in the 1930s to verify these propositions. Topos theory reformulates and generalizes set theory in a categorical setting, the objects in the categories of interest being akin to variable or parametrized sets. Such a category, called a topos, comes equipped with an internal logic which turns out to be strongly typed and intuitionistic. There are geometric aspects to