THEORY OF A MAGNETOHYDRODYNAMIC FLOW REGULATOR FOR LIQUID METAL

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A theory is developed for unsteady flow of liquid metal in a dc MHD flow regulator channel for small magnetic Reynolds numbers. It is shown that it is possible to use the quasi-stationary approximation for calculating the integral flow parameters.

MHD methods, along with their applications in direct transformation of thermal energy into electrical energy, in recent years have been ever more widely used in nuclear energy and metallurgy for transporting and for measuring the parameters of liquid metal flows [1, 2]. Complete sealing, operation over a wide temperature range, and simplicity of control and automation are among the unquestionable advantages of all MHD devices.

The existing theory of MHD devices is limited primarily to the stationary regimes of operation of electromagnetic pumps and magnetic flowmeters.

The most characteristic operating regimes for the MHD regulator are the transient regimes, which are defined primarily by the hydrodynamics of the liquid metal flow.

The present article is devoted to the study of the unsteady flow of liquid metal in the channel of a dc MHD flow regulator with independent excitation. The magnetic Reynolds are low ($R_m = \rho_0 U_0 b \ll 1$).

In the present case the MHD equations may be written in the form

\[
\begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\nabla p + \eta \mathbf{A} + \mathbf{j} \times \mathbf{B}, \\
\nabla \cdot \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\mathbf{j} &= \sigma \mathbf{E} + \mathbf{v}, \\
\nabla \cdot \mathbf{j} &= \nabla \cdot \mathbf{B} = 0,
\end{align*}
\]

(1)

Here $\mathbf{v}$ is the liquid metal flow velocity; $p$ is pressure; $\mathbf{B}$, $\mathbf{E}$ are the magnetic induction and the electric field intensity; $\mathbf{j}$ is the electric current density; $\rho$, $\eta$, $\sigma$, $\mu$ are, respectively, the density, dynamic viscosity, conductivity, and magnetic permeability of the liquid metal.

Let us consider the unsteady discharge of liquid metal from a reservoir through a long flat channel in the presence of a uniform magnetic field $B_0$ (Fig. 1). We shall assume that two channel walls $x = \pm a/2$ are thin and made from metal of low conductivity, and that the other two walls $x = \pm b/2$ are highly conductive electrodes which are closed across an external electrical circuit containing an emf source and the resistance $R_e$.

Under these conditions the equations in (1) reduce to the system

\[
\begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} &= -\nabla p + \eta \frac{\partial \mathbf{v}}{\partial z}^2 - j_y B_0, \\
\frac{\partial E_y}{\partial x} &= -\frac{\partial B_y}{\partial t}, \\
j_y &= -\mu \frac{\partial B_y}{\partial x}, \\
j_y &= \sigma (E_y + v B_0).
\end{align*}
\]

(2)

Integrating (2.2) with respect to $x$ and evaluating the terms of the resulting relation with respect to order of magnitude, we have

\[
E_y / U_0 B_0 \sim 1 \sim R_m b / T U_0 + f(t),
\]

where $U_0$ and $T$ are characteristic quantities (flow velocity and time).

For $R_m \ll 1$ and $b / T U_0 \ll 1$ we can assume that $E_y$ does not depend on $x$ and is a function only of time. Then, in accordance with the solution of the problem for the electric potential on the electrodes of the device, the intensity of the electric field in the regulator channel may be written in the form [3, 4]

\[
-U_y = \frac{1}{b} \int_{-b}^{b} v_0 \, dx, \quad k = \frac{R_e}{R_e + R_i}, \quad R_i = \frac{1}{\sigma} \frac{a}{b},
\]

(3)

Here $R_i$ is the internal electric resistance of the regulator of length $l$.

Allowing for (3) the momentum equation becomes

\[
\frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \frac{M_v}{b} \mathbf{G} + \eta \frac{\partial \mathbf{v}}{\partial z}^2 + \frac{M \sigma}{\rho} \mathbf{B} \cdot \mathbf{B} \cdot \frac{\partial \mathbf{v}}{\partial \mathbf{z}}^2 - \frac{M \sigma}{\rho} \mathbf{B} \cdot \mathbf{B} \cdot \frac{\partial \mathbf{v}}{\partial \mathbf{z}},
\]

(4)

with the boundary conditions

\[
\mathbf{v} = 0, \quad x = \pm a/2
\]

and the initial conditions

\[
v = 0, \quad t = 0
\]

At the initial instant of time the quantity $E_0$ or the resistance $R_e$ is varied, and the liquid metal begins to move.

We shall assume that the reservoir from which the metal discharges has finite dimensions, so that the level variation with time in the reservoir resulting from the metal flow cannot be neglected. However, we shall also assume that the pressure behind the reservoir is always equal to atmospheric. Under these conditions we can set

\[
P = P_0 - A \int_0^t U \, dt \\
A = \frac{S^*}{S} \frac{g}{U},
\]

(5)

Here $S^*$ and $S$ are the cross-sectional areas of the regulator channel and the reservoir, respectively.
We shall use the method of successive approximations to solve the posed problem.

Integrating (4) over the channel section, we obtain the following integrodifferential equation:

\[
\frac{dU}{dt} = P_0 - A \int_0^t U \, dt + \frac{Mv}{b} G + \frac{v}{b} \int_{-\gamma}^{\gamma} \frac{\partial^2 v}{\partial x^2} \, dx + \frac{Mv}{b^2} U(k-1). \tag{6}
\]

We shall assume that

\[
\int_{-\gamma}^{\gamma} \frac{\partial^2 v}{\partial x^2} \, dx = - \frac{U}{2b} K(\gamma, M, t) \approx - \frac{U}{2b} K(\gamma, M). \tag{7}
\]

Here the parameter \(K(\gamma, M)\) corresponds to the stationary flow regime \([5]\).

Then (6) becomes the following second-order differential equation:

\[
\frac{d^2 U}{dt^2} = N \frac{dU}{dt} - AU \left( N = \frac{-vK(\gamma, M)}{2b^2} + \frac{Mv}{b^2} (k-1) \right). \tag{8}
\]

To solve (8) we specify the following conditions:

\[
U \bigg|_{t=0} = 0, \quad \frac{\partial U}{\partial t} \bigg|_{t=0} = P_0 + \frac{Mv}{b} G. \tag{9}
\]

The solution of (8) has the form

\[
U(t) = c_1 e^{\lambda t} + c_2 e^{\lambda t} = \frac{1}{\lambda_1 - \lambda_2} \left( P_0 + \frac{Mv}{b} G \right). \tag{10}
\]

From the initial conditions it follows that

\[
c_1 = -c_2 = \frac{1}{\lambda_1 - \lambda_2} \left( P_0 + \frac{Mv}{b} G \right). \tag{11}
\]

From (5) we find

\[
P_1(t) = P_0 - A \times \left\{ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \left[ \frac{1}{\lambda_1} e^{\lambda_1 t} - 1 - \frac{1}{\lambda_2} e^{\lambda_2 t} - 1 \right] \right\}. \tag{12}
\]

We note that the first approximation solution contains the same qualitative features as the solution obtained for regulators of the induction type \([6]\). The expression (12) may be used to construct the second approximation. We shall use the Laplace transformation to solve (4); we introduce the notation

\[
V = \int_0^\infty \! e^{-st} \, dt, \quad \Pi = \int_0^\infty \! Pe^{-st} \, dt, \quad W = \int_0^\infty \! Ue^{-st} \, dt. \tag{13}
\]

Then we obtain from (4)

\[
\frac{d^2 V}{ds^2} - \left( \frac{a + \alpha}{s} \right) V = \frac{D(q)}{v}. \tag{14}
\]

The boundary conditions for (14) will be

\[
V = 0, \quad x = \pm \frac{1}{2} b. \tag{15}
\]

The solution of (14) will be the function

\[
V(x, q) = c_1 \text{ch} \sqrt{b} x + c_2 \text{sh} \sqrt{b} x + \frac{D(q)}{v \sqrt{b}} \left( \beta = \frac{q + \alpha}{s} \right). \tag{16}
\]

The integration constants are found from the boundary conditions

\[
c_2 = 0, \quad c_1 = -\frac{D(q)}{v \sqrt{b} \text{ch} \frac{1}{2} \sqrt{b} x}. \tag{17}
\]

Then the solution (16) takes the form

\[
V(x, q) = \frac{D(q)}{v \sqrt{b}} \left( 1 - \frac{\text{ch} \sqrt{b} x}{\text{sh} \frac{1}{2} \sqrt{b} x} \right). \tag{18}
\]

To exclude \(W(q)\) from the solution we use the relation

\[
W(q) = \frac{1}{b} \int_{-\gamma}^{\gamma} V(x, q) \, dx. \tag{19}
\]

As a final result we obtain

\[
V(x, q) = \left[ \Pi(q) + \frac{1}{q} G \right] b \sqrt{b} \theta(x, \beta) + \frac{b \sqrt{b} \theta(x, \beta)}{\theta(b, \beta)}, \tag{20}
\]

\[
\theta(x, \beta) = \text{ch} \frac{1}{2} \sqrt{b} x - \text{sh} \sqrt{b} x,
\]

\[
\theta(b, \beta) = \text{sh} \frac{1}{2} \sqrt{b} x \text{ch} \frac{1}{2} \sqrt{b} x - 2 \text{sh} \frac{1}{2} \sqrt{b} x.
\]

The Riemann-Mellin transform theorem yields

\[
v(x, t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \left[ \Pi(q) + \frac{1}{q} G \right] b \sqrt{b} \theta(x, \beta) \theta(b, \beta) e^{st} \, dq.
\]