One of the main results of the present paper is the representation of Eq. (1.2) in the form (4.7), which does not contain generalized functions. In the form (4.7), Eq. (1.2) becomes amenable to concrete numerical calculations.

It would be interesting to generalize the approach considered here to the relativistic case on the basis of quasipotential equations [12]. In this connection, we mention especially the three-dimensional formulation of the relativistic two-body problem given in [13].

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LITERATURE CITED


CLASSICAL RECOMBINATION CROSS SECTION

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Classical theory is used to calculate the recombination cross section of electrons in the field of an attractive Coulomb center. The cross section differs from the one calculated in accordance with Kramers's well-known expressions, which also has a semiclassical nature. It is shown that the results of the classical calculation and Kramers's formulas describe different cases. The classical recombination cross section applies for low initial velocities of the electrons, when emission of soft photons results in capture.

The calculations of the spectrum intensity of the radiation emitted by a charge moving along a hyperbolic trajectory in the field of a Coulomb center are well known [1]. The influence of the emission on the trajectory is not taken into account. Such influence is taken into account for the problem of finite motion in an attractive field (instability of the classical atom) and gives a sensible estimate for the lifetime of highly excited states in the hydrogen atom.

Since the emission of radiation reduces the energy E of the charge moving in the external field, in an attractive field one can have trajectories that in the limit \( t \to -\infty \) have hyperbolic asymptotic behavior.

(E > 0), and for \( t > t_0 \) describe finite motion (E < 0). Such trajectories exist for any value of the initial velocity \( v \) for impact parameters \( \rho \) less than some \( \rho_0 \). The quantity \( \sigma_c = \pi \rho_0^2 \), which describes the cross section for capture of the particle by the attractive Coulomb field is the classical recombination cross section.

We shall calculate this cross section as a function of the initial velocity \( v \). We consider the nonrelativistic case \( (v \ll c) \) and therefore restrict ourselves to the dipole approximation. The electron has charge \( e \) and mass \( m \) and the fixed Coulomb center the charge \(-Ze\). Suppose that in the limit \( t \to -\infty \) the electron has energy \( E_- \) and angular momentum \( M_- \):

\[
E_- = m v^2/2, \quad M_- = m p v.
\]

The energy and momentum depend on the time in accordance with the equations [1]

\[
dE/dt = -\beta^2 \eta r^{-4}(t),
\]

\[
dM/dt = -\gamma M/r^3(t),
\]

where \( \beta = Z e^2, \gamma = 2e^2/3m^2c^2 \), and \( r \) is the distance of the electron from the center. The function \( r(t) \) is determined by the equation

\[
(d/dt) \left( \frac{\beta^2}{m} \right) = \frac{2\beta}{mr} \left( \frac{E_-}{r^2} \right) + \frac{2\beta}{mr} \exp \left( -2\beta \right) \left( \frac{dt}{r^2} \right).
\]

It would hardly be possible to solve Eq. (4) in general form. We need it to estimate the conditions of applicability of the approximation that consists of the following. We note that for \( \rho = \rho_0 \) the asymptotic behavior of the trajectory as \( t \to +\infty \) is a parabola: the electron goes off to infinity with energy \( E_+ = 0 \) and angular momentum \( M_+ \). Reversing the sign of the time, we consider the change in the energy and angular momentum of a particle that falls toward a Coulomb center along a parabolic trajectory with angular momentum \( M_+ \) and increases its angular momentum and energy in accordance with Eqs. (2) and (3). Such a trajectory will have a final branch that is hyperbolic, and the value of the impact parameter \( \rho_0 \) will be determined by Eqs. (1). To calculate the total emitted energy

\[
E_- = \beta^2 \gamma \int_{-\infty}^{+\infty} \frac{dt'}{r(t')},
\]

we shall assume that the trajectory is parabolic for all \( t \). Then, using the parametric dependence between \( r \) and \( t \)

\[
r = \frac{p}{2} (1 + x^2) \quad t = \sqrt{\frac{mp^3}{r} \left( 1 + \frac{x^2}{3} \right)},
\]

where \( p = M^2/m \beta \), we have

\[
E_- = 2ne^2 p^2 m/Ze^2c.
\]

It can be seen from Eqs. (4) that the trajectory can be regarded as a parabola if the eccentricity of the orbit in the hyperbolic branch

\[
\varepsilon_- = \sqrt{1 + \frac{2E_- M_+^2}{m^2 \beta^2}}
\]

differs little in the limit \( t \to -\infty \) from unity \( (\varepsilon_- \approx 1) \) and if the change in the angular momentum during the motion is small:

\[
M_+ = M_- \exp \left( -\beta \gamma \int_{-\infty}^{+\infty} \frac{dt'}{r^2} \right) \quad M_+ \approx M_- < M.
\]

Both these conditions are satisfied if

\[
M_+ \approx 2^3 \varepsilon^2 / c.
\]

The inequality (6) also follows from general considerations: if it is violated, the motion of the electron near the center is essentially relativistic.

Allowance for the change in the trajectory due to the influence of radiation does not introduce significant changes into the result. Thus, assuming that the complete trajectory is an hyperbola with eccentricity \( \varepsilon_- \), for the total emitted energy we obtain the expression