INFLUENCE OF RELAXATION ON SELF-INDUCED TRANSPARENCY OF THE EXTRAORDINARY WAVE

G.T. Adamashvili

The inverse scattering method is used to investigate the influence of relaxation effects on the self-induced transparency in uniaxial molecular crystals in the presence of inhomogeneous broadening of a spectral line. The parameters of the extraordinary-wave soliton are determined.

1. Self-induced transparency occurs in a resonant medium under the influence of a light pulse whose intensity exceeds a certain threshold value and whose duration is less than the times of irreversible relaxation. When the effect occurs, there is a simultaneous decrease in the propagation velocity of such a pulse compared with the pulse velocity in the absence of resonance. In isotropic media, this phenomenon has been investigated both theoretically and experimentally (see, for example, the monograph [1] and the literature quoted there). In [2] there is a theoretical study of the effect in anisotropic media, in which so-called extraordinary waves [3] can propagate, in contrast to isotropic bodies. For the mathematical description of self-induced transparency for extraordinary waves it is necessary to specify equations for a system of noninteracting two-level molecules and the equation for the extraordinary wave. Simultaneous solution of this nonlinear system of equations yields the parameters of an extraordinary-wave $2\pi$ pulse (soliton).

An important achievement in the field of nonlinear equations and the theory of solitons was the development of the inverse scattering technique [4, 5]. Although many qualitative results for extraordinary-wave self-induced transparency have been obtained by simpler methods [2], appreciable progress is achieved by the use of the inverse scattering method. The advantage of the method is that it enables one to solve the Cauchy problem for the corresponding equations in the presence of inhomogeneous broadening of the spectral line.

Allowance for the relaxation effects leads to the appearance in the equations of additional small terms. At the same time, the inverse scattering method ceases to work, though it can provide a good basis for the development of perturbation theory [6-10]. In the present paper, in which we use perturbation theory, we consider the influence of relaxation on the extraordinary-wave soliton. Relaxation effects become important, of course, at pulse durations of the order of the relaxation times, and it should be noted that such a situation is frequently realized experimentally [1, 11]. Thus, the investigation is interesting and, undoubtedly, significantly extends the possibilities of experimental study of the effect in anisotropic crystals, which are frequently used in laser experiments.

2. We consider a uniaxial molecular crystal containing a small concentration of impurities whose optic axes coincide with the optic axis of the matrix. Such a case is realized for many laser crystals [12].

We assume that a plane wave propagates along the $z$ axis, which makes an angle $\theta$ with the optic axis $\xi$. We shall assume that the wave vector $k$ lies in the $yz$ plane; then $kr=k(z \cos \theta+y \sin \theta)=k \xi$, and the electric field intensity in the wave can be represented in the form

$$E^+(\xi, t)=\mathbf{g}^+(\xi, t)e^{i(\omega t-kz)}, \quad E^-(\xi, t)=\mathbf{g}^-(\xi, t)e^{i(\omega t+kz)},$$

where $\phi(\xi, t)$ is the phase function, and $\omega$ is the field frequency ($\omega \approx \omega_0$ is the excitation frequency of the two-level impurity). In the considered case under conditions of self-induced transparency a nonlinear connection between $E$ and the polarization $P$ of the impurity is realized only for the $z$ components of these vectors, the relations of linear crystal optics [3] holding for the remaining components. Taking into account the correction for the resultant field, we can represent the $z$ component of the polarization in the form

$$P_z^+(\xi, t)=idP^+e^{i(\omega t-kz)}+\frac{\text{fr}_1}{4\pi}E_z^+,$$

where $d$ is the transition dipole moment of the impurity.

The expressions (1) and (2) can be substituted in the wave equation [4]
\[
\frac{1}{c^2} \frac{\partial^2 E_x^+}{\partial t^2} \left( \frac{1 - \varepsilon_{\perp}}{\varepsilon_{\perp}} \cos \theta \right) \frac{\partial^2 E_x^+}{\partial z^2} = -4\pi \left( \frac{1}{c^2} \frac{\partial^2 P_x^+}{\partial t^2} - \frac{\cos \theta}{\varepsilon_{\perp}} \frac{\partial^2 P_x^+}{\partial z^2} \right),
\]
which is obtained from Maxwell's equations using the relations
\[
E_x = -\frac{\text{ctg} \theta}{\varepsilon_{\perp}} (E_x + i\lambda P_x), \quad P_x = \frac{\varepsilon_{\perp} - 1}{4\pi} \frac{\text{ctg} \theta}{\varepsilon_{\perp}} (E_x + i\lambda P_x),
\]
where $\varepsilon_{\perp} = \varepsilon_{\|} = \varepsilon_{p}$, $\varepsilon_{\|} = \varepsilon_{zz}$ are the components of the permittivity tensor of the matrix. Making the assumption that the amplitudes $E_x^+$, $p^+$ and the phase function $\phi$ vary slowly, we obtain in the presence of inhomogeneous broadening of the spectral line the equations
\[
\frac{\partial^2 E_x^+}{\partial t^2} + \frac{1}{c^2} \frac{\partial E_x^+}{\partial t} = \frac{\chi}{\nu_{\text{v}}} \int g(\Delta \omega) p^+(\Delta \omega) d\Delta \omega,
\]
where
\[
n^2 = \frac{\varepsilon_{\|} \varepsilon_{\perp}}{\varepsilon_{\perp} + (\varepsilon_{p} - \varepsilon_{\perp}) \cos^2 \theta},
\]
and $g(\Delta \omega)$ is the normalized function of the inhomogeneous broadening. The dispersion relation (5) determines the connection between the wave vector $k$ and the frequency $\omega$ of the extraordinary wave. The quantities $\rho^+$ and $N$ can be determined from Bloch's equations [1] ($N$ is the difference between the populations of the upper and lower levels of the impurity).

For the following, it is convenient to go over to the variables $X = \xi$, $\tau = t - \frac{\omega_0}{\nu_{\text{v}}}$. In them, the complete system of equations describing the extraordinary-wave self-induced transparency (the Bloch–Maxwell equation) has the form
\[
\frac{\partial \rho^+}{\partial \tau} = i\Delta \omega \rho^+-2rN\frac{\rho^+}{T_2}, \quad \frac{\partial N}{\partial \tau} = r\rho^--q\rho^+ + \frac{N-N_{eq}}{T_1},
\]
\[
\frac{\partial \tau}{\partial X} = r_x = \frac{\omega_0}{h\nu_{\text{v}}} \int g(\Delta \omega) \rho^+(\Delta \omega) d\Delta \omega,
\]
where $r = -(d/h)E_x^+ = -q^*$, $\Delta \omega = \omega_2 - \omega$, $\rho^* = (p^*)^*$, $N_{eq}$ is the equilibrium value of $N$, and $T_1$ and $T_2$ are the times of longitudinal and transverse relaxation.

3. We shall solve these equations using the inverse scattering method. In this method, an important part is played by the Zakharov–Shabat equations
\[
\frac{\partial u_1}{\partial \tau} = i\xi u_1 + q u_2, \quad \frac{\partial u_2}{\partial \tau} = i\xi u_2 + r u_1, \quad \int_{-\infty}^{+\infty} |q| d\tau < \infty, \quad \int_{-\infty}^{+\infty} |r| d\tau < \infty.
\]
These equations are investigated in [4, 5], and therefore, we shall not go into details but merely give here facts needed for what follows.

Suppose that $\Phi = \left( \Phi_1, \Phi_2 \right)$ and $\Phi = \left( \Phi_2^*, -\Phi_1^* \right)$ for real $\xi$ are the first pair of linearly-independent solutions of the Zakharov–Shabat equations and satisfy the asymptotic conditions
\[
\Phi \rightarrow \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{-\xi \tau}, \quad \Phi \rightarrow \left( \begin{array}{c} 0 \\ -1 \end{array} \right) e^{\xi \tau}, \quad \tau \rightarrow -\infty.
\]
From the asymptotic conditions as $\tau \rightarrow +\infty$
\[
\Phi \rightarrow \left( \begin{array}{c} a e^{-\xi \tau} \\ \tilde{b} e^{\xi \tau} \end{array} \right), \quad \Phi \rightarrow \left( \begin{array}{c} \tilde{a} e^{-\xi \tau} \\ b e^{\xi \tau} \end{array} \right)
\]
we can determine $a$, $b$, $\tilde{a}$, $\tilde{b}$ ($a \tilde{a} + b \tilde{b} = 1$). The second pair of linearly-independent solutions $\Psi = (\Psi_1, \Psi_2)$ and $\Psi = (\Psi_2^*, -\Psi_1^*)$ are related to $\Phi$ and $\Phi$ by