LITERATURE CITED


CALCULATION OF STATIC INTERQUARK POTENTIAL IN A STRING

MODEL IN A TIMELIKE GAUGE

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The quantum theory of a relativistic string with fixed ends is constructed using a timelike gauge. A gauge condition is introduced into the reparametrization-invariant string action. In the theory, there are no restrictions on the space-time dimension and there are no tachyon states. The constraints and gauge conditions are satisfied in the quantum case only on the average, at the level of the matrix elements between the physical state vectors. The string generates a static potential \( V(R) = \sqrt{2}R^2 + \varepsilon_0 \), where the constant \( \varepsilon_0 \) is a free parameter of the theory which can be determined from experiment.

1. Introduction

The model of a relativistic string [1,2] gives a transparent picture of quark confinement in hadrons. The string is a model of a tube of gluon field connecting the quarks [3,4]. It is very probable that such one-dimensional configurations of the gluon field are dominant at interquark separations around hadron diameters.

The string energy is proportional to its length, so that a relativistic string
connecting quarks leads to a potential that rises linearly with the distance. It can be shown that the static potential, i.e., the interaction potential of quarks at rest, can be consistently calculated in the framework of the string model in not only the nonrelativistic approximation [5] but also in relativistic quantum theory [6-10].

The most direct way to make such calculations is to investigate the model of a relativistic string with fixed ends (the approximation of infinitely heavy quarks). This problem was the subject of [11-13], in which the lightlike gauge, popular in the string model, was used. The use in this case of the same scheme of quantization as in the theory of a free relativistic string leads to the following difficulties [11,13]. The tachyon state in the theory of the free string is manifested in the considered problem as the first quantum correction to the linear potential generated by the string connecting the quarks. This correction has a negative sign and depends on the space-time dimension. It is important that this dimension cannot be arbitrary but must be equal to 26. Of course, this last result is difficult to reconcile with real hadron dynamics, although such attempts are made [13].

The unphysical dimension of space-time in the theory of a relativistic string with fixed ends using the lightlike gauge is a consequence of requiring the fulfillment at the quantum level of the Poincaré algebra or, more precisely, rotational invariance in the theory. However, it was correctly noted in [12] that in this problem there is, by virtue of its very formulation, no such invariance.

All these difficulties can be avoided if the static interquark potential generated by the relativistic string is calculated in a timelike gauge. The present note is devoted to this problem. The timelike gauge will be introduced directly in the parametrization-invariant action of the string.

The paper is arranged as follows. In the second section, we construct the Hamiltonian dynamics of the relativistic string with fixed ends in the timelike gauge. In the third section, we show how this model can be quantized without restrictions on the space-time dimension and without tachyon states. We also calculate the static interquark potential. In the Conclusions, we discuss briefly the obtained results. In the Appendix, we give a simple but consistent derivation within the nonrelativistic approximation of the linearly rising potential generated by the string.

2. Hamiltonian Dynamics of the Relativistic String in the Timelike Gauge

In the theory of the relativistic string, the lightlike gauge [1,2,14]

\[ n x = \frac{n P}{\gamma} \tau + n Q \]  

(2.1)

is popular; here, \( x^\mu(\tau, \sigma) \) are the string coordinates, \( n^\mu \) is a constant isotropic vector, \( n^2 = 0 \), which does not depend on \( \tau \) and \( \sigma \), \( P^\mu \) is the total momentum of the string, \( Q^\mu \) are the coordinates of the "center of mass" of the string at \( \tau = 0 \), and \( \gamma \) is a constant having the dimensions of the square of a mass. Its main advantage is that it enables one to solve in polynomial form the constraints on the string coordinates,

\[ (x^\pm \dot{x})^\gamma = 0, \]  

(2.2)

\[ \dot{x} = \frac{\partial x}{\partial \tau}, \quad \ddot{x} = \frac{\partial x}{\partial \sigma}, \]  

(2.3)

and express the dependent components of the radius vector of the world surface of the string in the form of quadratic combinations of the independent (transverse) components of this vector.

However, in the theory of a relativistic string with fixed ends this advantage of the lightlike gauge can be sacrificed and the physically transparent timelike gauge employed. This gauge is determined by Eq. (2.1) with \( n^2 = (n^\nu)(n^\nu) > 0 \). In the Lorentz frame in which \( n=0 \), the evolution parameter \( \tau \) is now proportional to the time \( t \).

Technically, it is convenient to introduce the timelike gauge directly in the parametrization-invariant action of the relativistic string:

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