THE GENERATION AND RADIATION OF THE SECOND HARMONIC IN THE NORMAL INCIDENCE OF AN ELECTROMAGNETIC WAVE ON AN INHOMOGENEOUS LAYER OF MAGNETICALLY ACTIVE PLASMA

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The radiation of the second harmonic from the region of plasma (hybrid) resonance in the normal incidence of an electromagnetic wave on a layer of inhomogeneous magnetically active plasma is investigated for the case in which the fraction of energy at the frequency of the incident wave that penetrates into the region of hybrid-resonance for the fundamental frequency is not exponentially small. Expressions are derived for the fields and the energy flux density of the second harmonic. It is shown that, just as in the case of an isotropic medium, the second harmonic is present only in the reflected wave, the second-harmonic field being proportional to the first power of the external magnetic field intensity.

As is well known, in the incidence of an extraordinary electromagnetic wave on a layer of inhomogeneous plasma, there is an abrupt growth of the amplitude of the electromagnetic field of the wave at the plasma-resonance point. Therefore in the region of plasma resonance it is primarily the nonlinear effects of the medium that appear. One such effect is the generation of an electromagnetic wave having a frequency equal to double the frequency of the incident wave. The generation of such a wave was considered in [1, 2] for the case of an isotropic plasma.

In the present paper the radiation of the second harmonic from the plasma-resonance region is investigated for normal incidence of an electromagnetic wave on a layer of inhomogeneous magnetically active plasma.

Let the density \( n_0 \) of the plasma when it is unperturbed by the wave field be a function of the space coordinate \( x \), such that \( \frac{dn_0}{dx} > 0 \) in the region where the wave field differs from zero, while the constant magnetic field \( H_0 \) is directed along the \( z \) axis. Then the electromagnetic field of the wave at the fundamental frequency \( \omega \) is conveniently represented in the form

\[
[E_1(x, t), H_1(x, t)] = 2\Re \left[ |E_1(x, H_1(x)| e^{-i\omega t} \right],
\]

where \( E_1(x) \) and \( H_1(x) \) are complex quantities. For the extraordinary wave the components of the electromagnetic field that are nonzero are \( E_x, E_y, \) and \( H_z \). The field component \( E_y \) satisfies the equation

\[
\frac{d^2E_y}{dx^2} + K^2(x) E_y = 0,
\]

where

\[
K(x) = \frac{\omega}{c} \left[ \frac{\Omega(1-\nu)}{1-\nu-\nu} \right]^{1/2},
\]

\( \nu = \Omega^2/\omega^2; \ u = \omega_H/\omega^2; \ \Omega \) is the plasma frequency of the electrons; \( \omega_H = eH_0/mc \) is the gyrofrequency of the electron.
the electrons \( e = -|e| \); the field component \( E_{1x} \) is related to \( E_{1y} \) by the following equation:

\[
E_{1x} = \frac{-(\mathbf{v}) \sqrt{\mathbf{u}}}{1 - \mathbf{u} - \mathbf{v}} E_{1y}. \tag{4}
\]

We shall assume that outside the neighborhoods of the reversal points \( (\mathbf{v} = \pm 1 \pm \sqrt{\mathbf{u}}) \) the inequality

\[
|K(x)| \gg 1
\]

is fulfilled, where \( 1/l = (1/n_0)(dn_0/dx) \) is the characteristic distance over which the plasma density changes. Then it follows that far from the reversal points and far from the point of plasma (hybrid) resonance, where \( 1 - \mathbf{u} - \mathbf{v} \approx 0 \), the solution of Eq. (2) may be sought by the WKB method. For the case when the reversal points lie close to the plasma-resonance points, which corresponds to fulfillment of the inequality

\[
\sqrt{\mathbf{u}} = \mathbf{u}/c \gg 1,
\]

the problem of the reflection of the extraordinary wave from a layer of magnetically active plasma was considered in the linear approximation in [3]. Since when inequality (16) is fulfilled the fraction of energy that penetrates into the region of plasma resonance is not exponentially small, in investigating the generation of the second harmonic we shall consider that condition (6) is fulfilled.

If the amplitude of the electric field of the incident wave in vacuo (the wave is incident from left to right) is represented in the form

\[
E_{1y}^{\text{Inc}}(x) = E_0 \exp \left( i \int_0^x K(x) \, dx \right),
\]

then, as follows from [3], in the neighborhood of the reversal points and in the neighborhood of the plasma-resonance point the field component \( E_{1y}(x) \) can be represented approximately as follows:

\[
E_{1y}(x) = A x + B [\gamma \ln x - 1/2],
\]

where \( x = 0 \) is the plasma-resonance point \( (\mathbf{v} | x = 0 = -u) \),

\[
A = \frac{\alpha}{\mathbf{v}} \mathbf{v} - i \mathbf{b}, \quad \mathbf{b} = \frac{\gamma}{\omega} \left[ \beta \mathbf{v} + i \mathbf{v} \right] \ll 1, \quad \gamma = \frac{d\mathbf{v}}{d\gamma} \bigg|_{x=0} \ll 1,
\]

\[
\alpha = \frac{u(1-u)}{\beta^{3/2}} \ll 1,
\]

\[
\mathbf{v} = \frac{3^{1/2} \Gamma(2/3)}{\Gamma(1/3)} - \frac{B}{a}, \quad B = i 2a - \frac{\alpha^{1/2} E_{1y}}{3^{1/3} \Gamma(2/3) \beta^{1/6}} e^{\alpha/4},
\]

\( \nu \) is the effective frequency of electron collisions with the remaining plasma particles; \( \Gamma(z) \) is the Euler \( \gamma \)-function.

It should be noted that the derived expressions are valid if in the region including the reversal points and the plasma-resonance point the dependence of the plasma density on the coordinate may be regarded as linear.

In order to obtain the equations for the components of the electromagnetic field at the second harmonic we represent \( \mathbf{v} \) (the velocity of the plasma electrons caused by the wave field), the wave-field components \( \mathbf{E} \) and \( \mathbf{H} \), and likewise the density of the plasma electrons \( n(x) \) in the form of series in powers of the field amplitude:

\[
\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \ldots, \quad \{\mathbf{E}, \mathbf{H}\} = \{\mathbf{E}_1 + \mathbf{E}_2 + \ldots, \mathbf{H}_1 - \mathbf{H}_2 + \ldots\}, \quad n(x) = n_0 + n_1 + \ldots
\]

Here the terms of the series having the subscript 1 correspond to the linear approximation.

Then from the equations of motion for \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) we obtain the following equations:

\[
\frac{\partial \mathbf{v}_1}{\partial t} - \mathbf{H} [\mathbf{v}_1 e_z] = \frac{e}{m} \mathbf{E}_1;
\]

\[
\frac{\partial \mathbf{v}_2}{\partial t} - \mathbf{H} [\mathbf{v}_2 e_z] = \frac{e}{m} \mathbf{E}_2 - (\mathbf{v}, \mathbf{v}) \mathbf{v}_1 + \frac{e}{mc} [\mathbf{v}_2, \mathbf{H}],
\]

where \( e_z \) is the unit vector in the direction of the external magnetic field.