The decay interaction of plasma and electromagnetic waves in a layer of homogeneous plasma is investigated. The characteristic parameters of the interaction are determined in the operating modes in which the amplification and generation of plasma oscillations occur. Estimates of the possibility of experimental observation of these effects are presented.

The stimulated combination scattering of transverse waves with a fixed phase in a plasma has been investigated in quite a large number of papers. The first investigations of the conversion of a pair of transverse waves into a longitudinal wave were undertaken simultaneously by a number of independent workers [1-3] in the approximation based on the specified field of transverse waves. Then in [4] the transformation coefficients and the characteristic interaction length were determined in the nonlinear approximation taking into account the back effect of the amplified waves on the pumping wave. However, in [4] only the conversion of the cotraveling transverse waves was investigated, although the interaction of oppositely traveling waves, as will be shown below, is more effective. Moreover, in the interaction of oppositely traveling waves, it is possible for generation of waves at combination frequencies to occur during the propagation of one pumping wave in a nonlinear layer,* whereas for the cotraveling waves only the amplification mode is realized (the amplitudes of the two waves must be nonzero). The present paper is devoted precisely to the study of the stimulated combination scattering of oppositely traveling electromagnetic waves in a plasma layer.

Let us consider the interaction of three waves satisfying the synchronism conditions

\[
\omega_1 - \omega_2 = \omega_p, \\
\mathbf{k}_1 - \mathbf{k}_2 = \mathbf{k}_p,
\]

in a homogeneous layer of isotropic plasma, in the absence of collisions. Here \(\omega_1\) and \(\mathbf{k}_1\) are the frequency and wave vector of the electromagnetic wave incident (in the +z-direction) on the layer; \(\omega_2\) and \(\mathbf{k}_2\) are the frequency and wave number of the oppositely traveling electromagnetic wave; \(\omega_p\) and \(\mathbf{k}_p\) correspond to the longitudinal plasma wave. In the solution we did not take into account the reflection of waves by the boundary layer, assuming the coefficients of reflection to be equal to zero. One of the possible realizations of such a system is a plasma layer having smooth variation of the electron concentration on the boundaries. Landau damping of the excited plasma wave may be neglected if the phase velocity of the plasma wave is large compared with the thermal velocity of the electrons. For transition of the wave into the region of low concentrations the Landau damping increases, and an effective boundary of the layer from which reflection of the plasma wave does not occur can be introduced. For transverse waves it is valid to neglect reflection if \(\omega_1, \omega_2 \gg \omega_p\).

*For example, in [5] the theory of the excitation of elastic waves was expounded; in [6, 7] the theory of excitation of acoustic waves was expounded. In [8] it was shown that parametrically coupled waves may be "captured" by the pumping: there exist oscillations the field of which is localized in the region occupied by the pumping radiation and, consequently, is amplified continuously.
The equation for the field in the plasma is
\[ \text{curl curl } E + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial}{\partial t} (j^{(1)} + j^{(2)}), \tag{2} \]
where \( j^{(1)} \) is the current that is linear in the field; \( j^{(2)} \) is the correction to it which is quadratic in \( E \). The solution of Eq. (2) is sought in the form of a sum of waves having slowly varying amplitudes:
\[ E = \sum_n a_n A_n \exp[i(\omega t - kz)]. \tag{3} \]
Using the expressions for the currents \( j^{(1)} \) and \( j^{(2)} \) (see, for example, [4, 9]), it is not difficult to obtain the well-known system of abridged equations for the amplitudes of interacting waves:
\[
\begin{align*}
\frac{dA_1}{dz} &= -\beta_1 A_1 A_2, \\
\frac{dA_2}{dz} &= -\beta_2 A_1^* A_1, \\
\frac{dA_2}{dz} &= \beta_1 A_1^* A_2 - \sigma A_1,
\end{align*}
\tag{4}
\]
in which we have taken into account the losses of the plasma wave associated with collisions or with Landau damping.* Here we have introduced the following notation:
\[
\begin{align*}
\beta_1 &= \frac{e}{2mc^2} \frac{\omega_p^2}{\omega_k^2} \frac{\omega_0}{\omega_1} \frac{k_1}{k_t}, \\
\beta_2 &= \frac{e}{2mc^2} \frac{\omega_p^2}{\omega_k^2} \frac{\omega_0}{\omega_1} \frac{k_1}{k_2}, \\
\beta_t &= \frac{e}{2mc^2} \frac{\omega_k^2}{\omega_0} \frac{c^2}{\Omega T_e^2}, \\
\sigma &= \frac{\nu}{V_{gr} t},
\end{align*}
\]
\( \omega_p^2 = 4\pi e^2 N/m; e \) and \( m \) are the charge and mass of an electron; \( N \) is the electron concentration; \( c \) is the velocity of light; \( V_{Te}^2 \) is the thermal velocity of the electrons; \( \nu \) is the effective collision frequency or the decrement of collisionless damping; \( V_{gr} t \) is the group velocity of the plasma wave.

The system of equations (4) is analogous to the equations that describe the interaction of three waves in nonlinear optics [5-8, 10]. We shall dwell on the determination of the characteristic parameters of back scattering in the plasma.

Let us first consider the solution of Eqs. (4) which satisfies the boundary conditions
\[ A_1(z = 0) = A_{10}, \quad A_2(z = L) = A_{2L}. \tag{5} \]
in the approximation based on the specified pumping field \( A_1(z) = A_1(0) = \text{const} \gg A_2(z), A_1(z) \). The coefficient of transformation of the pumping wave into an oppositely traveling electromagnetic wave (the nonlinear coefficient of reflection from a transparent plasma layer) is found to equal
\[
\left| \frac{A_2(0)}{A_{10}} \right|^2 = \frac{1}{|A_{10}|^2} \left| \frac{\Omega^2 + \frac{\omega_p^2}{4}}{\Omega \cos \Omega L + \frac{\omega_p}{2} \sin \Omega L} \beta A_{10} \right|^2,
\tag{6}
\]
where \( \Omega = \sqrt{\beta_1 \beta_2 |A_{10}|^2 - \omega_0^2/4} \). For \( \cot \Omega L = -\alpha/\Omega \) this quantity tends to \( \infty \), which means that there exists the possibility of the generation of waves at combination frequencies in the layer (obtaining nonzero solutions in the problem having the zero boundary conditions \( A_{10} = 0, A_{2L} = 0 \)). The self-excitation condition may be represented approximately (for estimation purposes) in the form
\[
\sqrt{\frac{V_{Te}^2}{6c^2} - \frac{\omega_p^2}{144 k_1^2 V_{Te}^2}} \approx \frac{\pi}{2}.
\tag{7}
\]
From this the threshold value of the pumping field can be estimated for stipulated parameters of the plasma layer. For example, for \( \nu/\omega_1 < 5V_{Te} \omega_{pe}/c^2 \)
\[
E_{\text{thr}} = \frac{\pi V_{Te}^2 cm \omega_{pe}}{2 \omega_{pe} L} \approx 1.4 \times 10^{-6} \frac{a_1}{L} \sqrt{\frac{T_e}{N^2}}.
\tag{8}
\]
*For \( \omega_1, \omega_2 >> \omega_{pe} \) the linear losses of the transverse waves may be neglected.