Sampling Design Optimization for Spatial Functions¹

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A new procedure is presented for minimizing the sampling requirements necessary to estimate a mappable spatial function at a specified level of accuracy. The technique is based on universal kriging, an estimation method within the theory of regionalized variables. Neither actual implementation of the sampling nor universal kriging estimations are necessary to make an optimal design. The average standard error and maximum standard error of estimation over the sampling domain are used as global indices of sampling efficiency. The procedure optimally selects those parameters controlling the magnitude of the indices, including the density and spatial pattern of the sample elements and the number of nearest sample elements used in the estimation. As an illustration, the network of observation wells used to monitor the water table in the Equus Beds of Kansas is analyzed and an improved sampling pattern suggested. This example demonstrates the practical utility of the procedure, which can be applied equally well to other spatial sampling problems, as the procedure is not limited by the nature of the spatial function.

KEY WORKS: spatial function, sampling, sample element, universal kriging, standard error, contour mapping.

INTRODUCTION

A spatial function is an association of numbers to a domain of geographic coordinates. Spatial functions may be one dimensional, such as a petrophysical well log, two dimensional, such as a contour map, or three dimensional, such as the variation in grade of ore within a mine. Spatial functions are continuous and uniquely defined over sizeable domains. Although fluctuations may be erratic and unpredictable in detail from one location to another, usually an underlying trend in the fluctuations precludes regarding the variable as completely random. Typically, observations which are closely spaced are autocorrelated.

Some spatial functions, for example, geothermal gradients, are not easily measured and their accurate characterization presents an expensive and time-consuming problem. Most spatial functions of a geologic nature can be known

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only partially through scattered sets of expensively gathered measurements. Observations of a spatial function constitute a statistical sample. However, because spatial functions possess continuity and each location is unique, classical statistical theory and sampling procedures are not applicable. Rather, we must turn to a special statistical theory which explicitly considers spatial properties, the theory of regionalized variables.

Although regionalized variable theory has been extensively described in the geomathematical and statistical literature, almost all discussions focus on either the problem of estimation or on simulation, while only a few studies have marginally addressed the question of the efficient arrangement of the sample elements (Alldredge and Alldredge, 1978; Ripley, 1981, p. 214-241). This paper treats the specific problem of sampling mappable geologic properties, the large class of regionalized variables whose observations can be regarded as points in two-dimensional space.

**OPTIMUM SAMPLING**

**Estimation Method**

The estimation method selected for this study is universal kriging, an unbiased linear estimator with minimum estimation variance properties, based upon the theory of regionalized variables (Matheron, 1971; Olea, 1975). Regionalized variable theory is a set of statistical principles which mathematically considers spatial function properties but which neglects the physical nature of the phenomenon under study. This makes the theory extremely general and accounts for its great range of applicability (Matheron, 1965; Journel and Huijbregts, 1978).

Regionalized variable theory uses random variables to model spatial functions. The standard deviation of an unbiased estimator of a random variable is the standard error (James and James, 1976). Since the standard error is given in the same units as the estimated value itself, it is useful as a measure of uncertainty in the estimate. A basic reason for selecting universal kriging is because it is a well-established estimation method for spatial functions that provides the standard error of the estimate.

**Measures of Sampling Performance**

The global performance of samples over the sampling domain of a spatial function can be judged by two indices: the average standard error and the maximum standard error. These measures depend upon:

1. Unmanageable factors
   a. The semivariance
   b. The drift