The Deterministic Side of Geostatistics

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The probabilistic approach is but one language used by geostatisticians to characterize spatial variability and to express a very simple criterion for goodness of estimation. Notions such as stationarity and ergodicity are important for the consistency of the probabilistic language but are irrelevant to the real problem, that of estimating a well-defined deterministic spatial average. The kriging algorithm is established without any recourse to probabilistic modeling or notation.

KEY WORDS: geostatistics, stationarity, ergodicity, spatial average, deterministic kriging.

INTRODUCTION

The probabilistic interpretation of a natural phenomenon known to be unique and the related hypothesis of stationarity do not always appeal to geologists' intuition, even though the positive aspects of geostatistical applications may be convincing. In fact the probabilistic approach and modeling are but one language (and possibly not the simplest) to express very simple deterministic criteria for estimation. Stationarity is not an intrinsic property of the deposit but is a property of the probabilistic model. A model choice and its properties must be judged on its efficiency in capturing and solving the problem at hand. Engineers are correct in judging geostatistics by efficiency and practical records rather than by the intricacy of its theoretical developments; if there existed a simpler language leading to the same algorithms and results there is little doubt that it would be adopted.

Probabilistic hypotheses such as stationarity and isotropy are shown to be equivalent to an experimenter's decision to look at average spatial characteristics over particular subareas and/or directions. Kriging is shown to be equivalent to a deterministic process of averaging errors of a same type over a predefined field where such averaging makes physical (geological) sense.

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Stationarity: A Model Choice

The probabilistic inference of a phenomenon requires some repetitiveness of the phenomenon observed. Natural phenomena are rarely as repetitive as the rolling of a die could be. In the earth sciences repetitiveness is usually obtained by moving within a certain area or volume. For example, consider a copper deposit $D$, $z(x)$ being the copper grade at point $x \in D$. The population $D$ includes several heterogeneous subpopulations: oxide and sulfide mineralizations, each with various degrees of alterations, rock types, etc. If the mean grade over $D$ is required, repetitiveness is considered over the whole deposit $D$, care being given to represent evenly each subpopulation or mineralization. The grade assessment may also be done per type of mineralization; repetitiveness is then obtained by moving the location $x$ within that type.

The definition of the population within which repetitiveness is obtained is a decision of the experimenter, for no two individuals are exactly identical and the initial large population can be split into as many subgroups as there are individuals. That decision of the experimenter is guided by various considerations:

(i) The chemistry (or physics) of the phenomenon studied: oxide and sulfide Cu mineralizations should be considered separately not only because they will require different milling processes, but also because they correspond to clearly different geological geneses. Conversely, for sulfide mineralization, it is not essential to separate bornite from chalcopyrite mineralization.

(ii) The amount and type of information available: if many populations are considered, many more data are needed to characterize each of them. At an early stage of exploration, the geologist-geostatistician has no choice but to consider a single population; as more information becomes available, he/she can start separating mineralizations. The distinction between bornite (very high Cu grade) and chalcopyrite may always remain inaccessible if data are defined on a large support, such as a core of 5 m length; moreover such distinction is irrelevant because no mining or milling will be able to separate these two sulfide mineralizations.

In probabilistic terms, the area or subpopulation within which repetitiveness is assumed corresponds to a "stationary" area or subpopulation. More precisely, the spatial variability of the phenomenon $z(x)$, where $x$ is in the area $A$, is modeled by a stationary random function \{Z(x), x \in A\}. A random function $Z(x)$ is a stationary if its multivariate distribution is invariant by translation within the space or area $A$, entailing that all its moments are also invariant by translation; see Doob (1953).

Stationarity allows statistical inference, for example

(1) since $\text{Prob} \{Z(x) \leq z\} = F(z)$, for all $x \in A$, the distribution function $F(z)$ can be estimated by the cumulative histogram of data $z(x_\alpha)$ taken at different locations $x_\alpha \in A$. 