where $\kappa = \kappa(\alpha, g)$ is defined by Eq. (28) in [1]. The first correction to the level $-\kappa^2$ is

$$
\mathcal{G}^{(1)} = \int d^3r \psi^*_b \nabla^2 \psi_{bd} = -\frac{\pi e^4 B^2}{12 \kappa^2 (\pi + g \kappa^2)}.
$$

4. The result of the present paper is the formulation and construction of the total Hamiltonian for the problem of the motion of a particle in an electromagnetic field in the presence of a strongly singular potential. The Hamiltonian is given by Eqs. (8), (7), and (4). The results can be used to find the perturbation-theory corrections to the levels obtained in [1]. We emphasize that the von Neumann extension procedure cannot in practice be applied to this problem, since this would require exact solutions of the equations $[-\Delta_{\alpha} + e A_{\alpha}(r)] \psi = \pm i \psi$ for a particle in an external field of general form.

**LITERATURE CITED**


**ENERGY LEVELS OF AN OSCILLATOR WITH SINGULAR CONCENTRATED POTENTIAL**

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The algebra of generalized functions constructed earlier by Shirokov is used to solve the Schrödinger equation for a harmonic oscillator on which a singular concentrated potential acts. The small corrections to the energy levels due to this potential are obtained.

1. In [1], one of the present authors obtained a renormalized algebra of local observables for one-dimensional quantum mechanics in which one can have potentials with singularities at a fixed point $x = 0$ of the type $\delta^{(n)}(x)$ for $n = 0, 1, 2, \ldots$. It was shown that the corresponding state vectors can also have singularities of the same type. In [2], a transition was made to more restricted algebras, for which the bilinear form is positive, so that Schrödinger equations with strongly singular concentrated potentials acquire physical meaning.

2. In the present paper, we consider the Schrödinger equation for a harmonic oscillator on which a singular concentrated potential acts at the point $x = 0$. We write down the Hamiltonian of this equation in the notation introduced in [1]:

$$
H_{\psi} = -\left( \frac{d}{dx} + \frac{1}{4} x^2 \right)^2 + \frac{1}{4} x^2
$$

or

$$
H_{\psi}(x) = -\psi''(x) - g \delta(x) \psi''(x) + 2 \delta(x) \psi'(x) + 2 \delta'(x) \psi(x) + g \delta''(x) \psi(x) + \frac{1}{4} x^2 \psi(x).
$$

The Hamiltonian (1) acts on state vectors of the type [1]

$$
\psi(x) = \psi_+(x) + \varepsilon(x) \psi_0(x) + g \delta(x) \psi_0(x),
$$

where

$$
g > 0, \quad \psi_+ = \frac{1}{2} (\psi_+ + \psi_-), \quad \psi_0 = \frac{1}{2} (\psi_+ - \psi_-), \quad \frac{d^2}{dx^2} \psi_0(\pm 0) = \psi_0(x).
$$

In the space $\{\psi\}$ of state vectors of the type (3) the Hamiltonian $H$ is not self-adjoint but merely the adjoint of a symmetric operator. The Hamiltonian becomes self-adjoint if the space $\{\psi\}$ is restricted to the
subspace \( (\psi)_n \), in which the state vectors satisfy the relations (36) of [2]:

\[
\psi_n = a_{n_0} \psi_0 + a_{n_1} \psi_1', \quad \psi_n' = a_{n_1} \psi_1 + a_{n_2} \psi_2' + g \psi_n',
\]

where \( \psi_n(0), \psi_n'(0), \psi_0(0), \) etc. The coefficients \( a_{n_0} \) satisfy the conditions

\[
a_{n_1} = -a_{n_0}, \quad a_{n_1} = a_{n_0}^*, \quad a_{n_2} = a_{n_1}^*.
\]

We shall obtain all the eigenfunctions and eigenvalues of the Hamiltonian \( H \) in the space \( (\psi)_n \). We seek the solution in the form

\[
\psi(x) = D_\nu(-x) \quad \text{for } x < 0, \quad \psi(x) = A_\nu D_\nu(x) \quad \text{for } x > 0,
\]

where \( A_\nu \) is a constant, \( D_\nu(x) \) is a parabolic cylinder function, and \( \nu = E - \frac{1}{2} \). This follows from the fact that the parabolic cylinder functions, and no other functions, satisfy the Schrödinger equation for a harmonic oscillator with \( E = \nu + \frac{1}{2} \) and decrease rapidly as \( x \to \pm \infty \).

The parabolic cylinder functions and their derivatives at the point \( x = 0 \) have the values (see, for example, [3])

\[
D_\nu(0) = -2 \frac{\nu}{\Gamma(-\nu/2)} \left( 1 + a_{n_0} a_{n_1} + a_{n_0} a_{n_2} + ga_{n_2} \left( \nu + \frac{1}{2} \right) \right), \quad D_\nu'(0) = -\frac{\nu}{\Gamma(-\nu/2)} \left( 1 + a_{n_0} a_{n_1} + a_{n_0} a_{n_2} + ga_{n_2} \left( \nu + \frac{1}{2} \right) \right),
\]

where \( F(\nu - \frac{1}{2}) \) and \( F(-\nu/2) \) are gamma functions of the corresponding arguments. Substituting these values in (7), we find from the boundary conditions (5) that they are satisfied only for values of \( \nu \) satisfying the equation

\[
1 = \frac{1}{2} \frac{\nu}{\Gamma(-\nu/2)} \left( 1 + a_{n_0} a_{n_1} + a_{n_0} a_{n_2} + ga_{n_2} \left( \nu + \frac{1}{2} \right) \right) \frac{\nu}{\Gamma(-\nu/2)} = 0.
\]

For the constant \( A_\nu \) in (7) for each \( \nu \) satisfying (9) we obtain

\[
A_\nu = -2 a_{n_0} \left( 1 + a_{n_0} + \frac{\nu}{\Gamma(-\nu/2)} \left( 1 + a_{n_0} a_{n_1} + a_{n_0} a_{n_2} + ga_{n_2} \left( \nu + \frac{1}{2} \right) \right) \right).
\]

For \( a_{n_0} = a_{n_1} = 0 \), Eq. (9) has a system of roots corresponding to the poles of the function \( \Gamma(-\nu/2) \), i.e., \( \nu = 2k \), where \( k = 0, 1, 2, \ldots \). For \( a_{n_0} = 0 \), this equation has a system of roots corresponding to the poles of the function \( \Gamma(1/2 - \nu/2) \), i.e., at \( \nu = 2k + 1 \). All the listed roots correspond to levels and eigenfunctions of the unperturbed oscillator. The existence of such roots is due to the circumstance that the potentials described by the constants \( a_{n_0} \) and \( a_{n_1} \) act only on even states, while the potentials described by the constant \( a_{n_2} \) act only on odd states.

In the limit \( \omega \to 0 \) (\( \omega \) is the eigenfrequency of the oscillator) we obtain from (9) Eq. (59) of [2], which determines the energy levels of the bound states of a particle moving in the field of the singular compensated potential (55) of [2].

3. We find the small correction to the harmonic oscillator energy due to a singularity of the type (1) at the point \( x = 0 \). For this, we consider values \( \nu = n + \alpha \), where \( |\alpha| \ll 1 \). In the case \( n = 2k \) \((k = 0, 1, 2, \ldots)\), the first correction in \( \alpha \) has the form

\[
\alpha_{2k} = \left[ a_{n_0} - g \left( 2k + \frac{1}{2} \right) \right] \left[ g + \frac{\nu}{2} \frac{2^\alpha(\alpha/k)^k}{(2k)!} \left( 1 + a_{n_0} a_{n_1}^* + a_{n_1} a_{n_2}^* - g a_{n_2} \left( 2k + \frac{1}{2} \right) - a_{n_3} \right) \right]^{-1}.
\]

Here, \( \psi(1/2 - k) \) is the logarithmic derivative of \( \Gamma(1/2 - k) \). For \( n = 2k + 1 \),

\[
\alpha_{2k+1} = -2 a_{n_0} \left( 2k + 1 \right) \left[ g + \frac{\nu}{2} \frac{2^\alpha(\alpha/k)^k}{(2k)!} \left( 1 + a_{n_0} a_{n_1}^* + a_{n_1} a_{n_2}^* - g a_{n_2} \left( 2k + \frac{3}{2} \right) \right) \right] + 2 a_{n_0} \left( 2k + 2 \right) \psi \left( \frac{1}{2} - k + 1 \right) \right]^{-1}.
\]

For the ground-state level \( (n = 0) \), the correction is

\[
\alpha_0 = \frac{2 a_{n_0} - g}{2g + \frac{\nu}{2} (2a_{n_0} a_{n_1}^* + a_{n_1} a_{n_2}^* - ga_{n_2}) - (2 - 2a_{n_0}) (C + 2 \ln 2)}.
\]