A rigorous solution is obtained to the problem of diffraction of H-polarized plane waves by a "louver"-type ribbon grating located above an ideally conducting shield. The characteristic features of the scattered field are analyzed. Particular attention is paid to the effect of such a shield on the diffractive properties of the grating.

1. Formulation of the Problem

We consider the problem of diffraction of H-polarized plane waves by a periodic structure which consists of ideally conducting infinitesimally thin ribbons of equal widths (Fig. 1) and is located at a distance $a$ above an ideally conducting shield.

The planes of the ribbons form an angle $\psi$ with the normal to the plane of the grating. This is a model problem for the problem of distorted radar characteristics of objects [1].

The method which we will use here is based on the results of earlier studies [2, 3] and allows reducing the given problem to two infinite systems of linear algebraic Fredholm equations of the second kind with matrix elements which decrease exponentially with higher indexes. This latter feature makes it possible to apply the truncation procedure in a numerical analysis of the problem.

We represent the incident wave in the form $H_x = \exp[ik(\gamma \sin \varphi - z \cos \varphi)]$ and require that the sought scattered field satisfy the condition of a finite energy integral within any bounded volume as well as the condition of radiating at infinity, both these conditions ensuring a unique solution to the given diffraction problem [2]. It is not difficult to demonstrate that the thus formulated problem reduces to the problem of determining the sets of amplitude coefficients $\{a_n\}_{n=-\infty}^{\infty}, \{b_n\}_{n=-\infty}^{\infty}$ and $\{c_m, d_m\}_{m=-\infty}^{\infty}$ in the Fourier representation of the field scattered by the grating in the respective regions $z \geq h \cos \psi - a - h \cos \psi \leq z \leq h \cos \psi$ and $-h + l \sin \psi \leq z \leq h$, $0 \leq \gamma \leq l \cos \psi$, which coefficients belong in the space of infinite sequences $l_2 = \{a: \sum_{n} |a_n|^2 (1 + |n|) < \infty\}$ and satisfy the infinite system of simultaneous linear algebraic equations of the first kind

$$\sum_{n=-\infty}^{\infty} \frac{a_n}{\Gamma_n^\delta + \omega_q} = -\frac{\tilde{d}_q}{\Gamma_q^\delta - \omega_q} ;$$

(1a)

Fig. 1. Geometry of the problem.

We have introduced here the following notation:

\[ \tilde{a}_n = -\frac{\Phi_n^+}{\Phi_0} a_n, \quad \tilde{b}_n = \frac{\Phi_n^-}{\Phi_0} b_n, \quad Y_n = \frac{\Phi_n^+}{\Phi_n^-}, \]

\[ \Phi_n^- = \Gamma_n \sin \phi \pm \Phi_n \cos \phi, \quad \Gamma_n^+ = \Gamma_n \cos \phi \pm \Phi_n \sin \phi, \]

\[ \Gamma_n = \sqrt{x^2 - \Phi_n^2}, \quad \Phi_n = \kappa + x \sin \phi, \quad E_n = \exp\left(i \Gamma_n \frac{4\pi}{l} a\right), \]

\[ x = \frac{l}{\lambda}, \quad k = \frac{2\pi}{\lambda}, \quad \vec{d}_q = V^+_q d_q, \quad \vec{c}_q = V^-_q c_q \exp(-2\pi i \omega_q \sin \phi), \]

\[ V^+_q = \frac{2\pi i \omega_q \cos \phi A_{\omega_0}}{\exp[2\pi l (\pm \omega_q \sin \phi + \Phi_0)] (-1)^q + 1 \Phi_0^-}, \]

\[ A_{\omega_0} = \begin{cases} 2 & (q = 0) \\ 1 & (q \neq 0) \end{cases}, \quad \omega_m = \sqrt{x^2 - \left(\frac{m}{2 \cos \phi}\right)^2}, \quad c_q = \exp(4\pi i \omega_q \phi), \]

\[ \delta = \frac{\hbar}{l} \quad \text{and the sign of } \sqrt{\Lambda} \text{ is selected so that } \text{Im} \sqrt{\Lambda} > 0 \text{ but that } \text{Re} \sqrt{\Lambda} \geq 0 \text{ when } \text{Im} \sqrt{\Lambda} = 0. \]

2. **Rigorous Solution in the Form of Infinite Systems of Equations of the Second Kind**

The infinite system of the first kind (1) belongs in the class of ill-conditioned ones.

The ill-conditioning of such systems is due to the properties of matrix operators (irreversibility etc.) and is usually manifested in a weak stability of the solutions to the corresponding finite-dimensional systems as well as in their nonconvergence to an exact solution as more equations are included before truncation of the system by any method [2, 4, 5]. Nevertheless, this system can be reduced to a form amenable to an effective analytical and numerical treatment of the problem. Reducing the system to such a form is possible by means of regularization, i.e., inversion of the principal part of matrix operators whose elements decrease slowly with higher indexes.

It is quite evident from expressions (1) that the principal part of the matrix operator in the second and the third subsystems here is an operator with a matrix kernel of the \((\Gamma_n^+ - \omega_q)\) kind in the denominator. The apparatus of analytical inversion of such matrix operators has already been sufficiently well developed in the theory of periodic gratings, where the method of partial inversion was applied to the rigorous solution of the problem of wave scattering by "louver"-type ribbon gratings. For equivalent transformations in this case here one must know the solution to the infinite systems of equations

\[ \sum_{n=-\infty}^{\infty} \frac{\gamma_n^+}{\Gamma_n^+ - \omega_q} = \frac{1}{\Gamma_n^+ + \omega_q} \quad (q = 0, 1, 2, 3, \ldots; \quad s = 0, \pm 1, \pm 2, \ldots); \]

\[ \sum_{n=-\infty}^{\infty} \frac{\gamma_n^+}{\Gamma_n^+ - \omega_q} = \gamma_0^+ \quad (p = 0, 1, 2, \ldots). \]