DIFFRACTION OF A $H_{10}$ WAVE AT A DIELECTRIC STEP

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A projection method is used in deriving a rigorous solution for the diffraction of a $H_{10}$ waveguide wave in an unbounded rectangular waveguide on encountering a semi-infinite dielectric step along the side wall. The relationship of some of the diffraction characteristics to the system parameters is examined.

INTRODUCTION

Many waveguide devices involve the use of various dielectric fillings; interest attaches to the case where an empty guide adjoins a guide partially filled with an insulator along the side wall, both guides being rectangular and of identical cross section. An $H_{10}$ wave is the basic wave type that can propagate in the regions to both sides of the junction. We determined the physical characteristics of the system when an $H_{10}$ wave is incident on the junction from the empty side, which is assumed to contain only one mode.

This problem has been partly solved elsewhere [1], where a variational method was used to reduce the problem to solution of an infinite system of linear homogeneous algebraic equations, which can be solved by reduction. However, no numerical results on the subject have so far been published.

Here a projection method is used to reduce the electrodynamic problem to an infinite system of linear inhomogenous algebraic equations of the second kind, which can be solved by reduction and iteration.

Various forms of projection methods have been used to solve many different waveguide problems (see the survey of [3]). Our form of projection method is essentially analogous to that of [4], since here the basic functions are two groups of functions orthogonal over a given interval, in contrast to other forms of the method. Consequently, the problem can be reduced to that of solving an infinite system of linear algebraic equations of the second kind, which is an undoubted advantage [5] over systems of the first kind.

1. FUNCTIONAL EQUATIONS

We now give the mathematical formulation. Consider an unbounded rectangular waveguide with ideally conducting walls in which the side wall bears a semi-infinite ideal dielectric plate of thickness $d$ with absolute constants $\varepsilon$ and $\mu_0 = 4\pi 10^{-7} \text{H/m}$ (Fig. 1). We assume that at the interface ($Z = 0$) there is an $H_{10}$ wave incident along the positive direction of the $Z$ axis, which arrives from the empty guide and strikes the dielectric step and which has components

![Fig. 1](image-url)
\[ E_y = \sin \left( \frac{\pi x}{a} \right) \exp \left( -i k z + i \omega t \right) (\text{Im} \ h_1 < 0). \]  

We have to find the scatter field arising from diffraction of the wave of (1) at the step.

The field components in the various parts of the guide can be written as

\[ E_{y1} = E_y + \sum_{m=1}^{\infty} A_m \sin \frac{\pi mx}{a} \times \exp \left( i h_m^2 x + i \omega t \right) (z < 0); \]  

\[ E_{y2} = \sum_{m=1}^{\infty} B_m \Phi_m(x) \times \exp \left( -i h_m^2 x + i \omega t \right) (z > 0) \]

\[ (\text{Im} h_m < 0, \ \text{Im} h_m' < 0), \]

where \( h_m = \sqrt{k_0^2 - (\pi m/a)^2}; \) \( k_0 \) is the free-space wave number; \( a \), waveguide width; and \( h_m' \), longitudinal wave number for an \( H_{m0} \) wave in a waveguide containing a dielectric, which is the solution to the following transcendental equation [1, 2]:

\[ \tan \left( \sqrt{k_0^2 - h_m'^2} \cdot dz \right) = \frac{1}{i}; \]

\[ \Phi_m(x) = \begin{cases} \sin q_m d \sin \frac{p_m(a - x)}{m} & \text{for } d \leq x < a, \\ \sin q_m x & \text{for } 0 \leq x \leq d \end{cases}, \]

\[ q_m = \frac{\left( k_0^2 - h_m'^2 \right)^{1/2}}{p_m} = \left( q_m^2 + k_0^2 - k^2 \right)^{1/2}. \]

Here \( k = \omega \sqrt{\varepsilon_0} \); the sequence of coefficients \( A_m \) and \( B_m \) is to be determined.

We use the conditions for linkup of the fields at \( Z = 0 \) to get the following system of functional (summator) equations for the \( A_m \) and \( B_m \):

\[ \sum_{m=1}^{\infty} \left( A_m \sin \frac{\pi mx}{a} - B_m \Phi_m(x) \right) = - \sin \frac{\pi x}{a}, \]

\[ \sum_{m=1}^{\infty} \left( A_m h_m \sin \frac{\pi mx}{a} + B_m h_m' \Phi_m(x) \right) = h_1 \sin \frac{\pi X}{a}. \]

The coordinate functions appearing in this system form an orthogonal system in the interval \([0, a]\), and the following relations apply:

\[ \int_0^a \sin \frac{\pi mx}{a} \sin \frac{\pi nx}{a} \, dx = \frac{a}{n} \delta_{mn}; \]

\[ \int_0^a \Phi_m(x) \Phi_n(x) \, dx = \frac{a}{2} R_m \delta_{mn}. \]

where

\[ R_m = \frac{d}{a} + \frac{\left( 1 - \frac{d}{a} \right) \sin^2 q_m d}{\sin^2 \left( (a - d) \sqrt{q_m^2 - \gamma^2} \right)} + \frac{\gamma^2 \sin^2 q_m d}{2a q_m (q_m^2 - \gamma^2)}. \]

Conditions (7) and (8) allow us to transform the system of functional equations to an infinite system of linear inhomogeneous algebraic equations either for the \( A_m \) or for the \( B_m \). For example, the system for \( A_m \) takes the form

\[ A_n = b_n - \sum_{m \neq n} A_m Z_{mn} \]

\[ (n = 1, 2, 3, \ldots). \]