The interaction of plane electromagnetic waves propagating in opposition is discussed. Scattering of the waves not only at the first spatial harmonic of the electron temperature, but also at succeeding ones, is taken into account. Equations are derived for the field amplitudes of waves of arbitrary power.

It is necessary in the case of normal incidence of a powerful radio wave on the ionosphere to take the presence of the reflected wave into account. Actually, the incident and reflected waves form a standing wave, due to which a spatial periodic structure of the electron temperature arises in the ionosphere by which both waves are in turn scattered. This phenomenon has been discussed in [1] in the case of weak nonlinearity of the medium, when the spatial distribution of the electron temperature can be assumed to be sinusoidal and the field of the wave can be represented in the form

\[ E = E_p \left[ a, \exp{(i \omega t - ikz)} + a^*, \exp{(i \omega t + ikz)} + c.c. \right]. \]

Strictly speaking, the electron temperature distribution has a more complex form, and one should take account not only of its first spatial harmonic, but also of the succeeding ones. Then the solution of the wave equation

\[ \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial}{\partial t} \left( |E|^2 \right) \frac{\partial E}{\partial t} - \frac{4 \pi}{c^2} \varepsilon_0 \frac{\partial E}{\partial t} = 0 \]  

can be sought in the more general form

\[ E = E_p r \cos(\omega t - \varphi), \]

where \( r \) and \( \varphi \) are some functions of the coordinate \( z \) [2]. In this case Eq. (1) is transformed into the system

\[ 0 \dot{y} \dot{y} - (y) = -M^2 + 4 \varepsilon_0 \gamma = -\nu y^2 f(y), \]

where \( y = r^2, \dot{y} = \frac{\omega}{c} r, M = m \frac{d}{d^2}, \nu = \frac{4 \pi \gamma_0}{\omega} = \frac{\omega_0^2}{\omega^2 + \omega_0^2}, \gamma = \frac{\omega_0^2}{\omega^2 + \omega_0^2}, g(\nu) = \frac{\nu_0 (\omega^2 + \nu_0^2)}{\omega^2 + \omega_0^2}, \text{ and } \varepsilon_0 \) is the permittivity of the unperturbed plasma. The dot above a letter denotes differentiation with respect to \( \theta \), and the dependence of \( r \) on \( z \) is determined by the equation [3]

\[ \frac{T_r}{T} = 1 = r^2 \frac{\omega^2 + \nu_0^2}{\omega^2 + \varepsilon_{eff}(T_r)}, \]

where \( \nu_{eff} = \nu_0 (\Omega_e / T) / \gamma = \nu_0 r \) and \((1/2) \leq \gamma \leq 1\).

We will assume that \( \mu \ll 1 \). The solution of Eq. (3) when \( \mu = 0 \) has the rather simple form

\[ y = a + b \cos 2 \sqrt{\varepsilon_0} \theta, \]

\[ M = M_0 = V \varepsilon_0 \sqrt{\varepsilon^2 - b^2} = \text{const}. \]

Actually, in a medium without absorption (\( \mu = 0 \)) it is possible to write the field of waves propagating in opposition as

\[ E = \alpha \cos(\omega t - kz) + \beta \cos(\omega t + kz), \]

which agrees with Eq. (5) if we set
\[ z^2 - \varphi^2 = a, \quad 2x^2 = b, \quad kz = \frac{\omega}{c} = \sqrt{\frac{z}{b}}. \] (6)

We will seek a solution of Eq. (3) for \( \mu \neq 0 \) according to [4] in the form of the expansion
\[ y = a + b \cos \varphi + \mu a_1(a, b, \varphi) + \mu^2 a_2(a, b, \varphi) + \ldots, \]
\[ M = \frac{M_0(a, b) + \mu M_1(a, b, \varphi) + \mu^2 M_2(a, b, \varphi) + \ldots}{M_0}, \] (7)

where
\[ \frac{d a}{d t} = \mu A_1(a, b) + \mu^2 A_2(a, b) + \ldots, \]
\[ \frac{d b}{d t} = \mu B_1(a, b) + \mu^2 B_2(a, b) + \ldots, \]
\[ \frac{d \varphi}{d t} = 2 \sqrt{z} + \mu D_1(a, b) + \mu^2 D_2(a, b) + \ldots. \]

Here \( u_1(a, b, \psi) \) and \( u_2(a, b, \psi) \) are periodic functions of the angle \( \psi \) with period \( 2\pi \).

Substituting Eqs. (7) and their derivatives into the original system (3) and collecting terms of the same powers in the parameter \( \mu \), we obtain the system
\[ \left( \frac{\partial^2 u_k}{\partial \varphi^2} - \frac{B_1}{V_{z_0}} \sin \varphi - b \frac{D_1}{V_{z_0}} \cos \varphi \right) (a + b \cos \varphi) + b \sin \varphi \left( \frac{\partial u_k}{\partial \varphi} + \frac{B_2}{2 V_{z_0}} \cos \varphi \right) \]
\[ + \frac{B_1}{2 V_{z_0}} \cos \varphi \sin \varphi - \frac{A_1}{2 V_{z_0}} b \frac{D_2}{V_{z_0}} \sin \varphi \right) \right) \frac{M_0}{z_0} \log \frac{1}{\mu} + \frac{a u_k}{8 z_0} = \frac{1}{8 z_0} (a + b \cos \varphi)^2 f(a + b \cos \varphi), \] (8)

Let us represent the right-hand sides of (8) in the form of Fourier series:
\[ (a + b \cos \varphi)^2 f(a + b \cos \varphi) = \frac{F_0}{2} + \sum_{n=1}^{\infty} F_n \cos n \varphi, \]
\[ (a + b \cos \varphi) g(a + b \cos \varphi) = \frac{G_0}{2} + \sum_{n=1}^{\infty} G_n \cos n \varphi. \] (9)

We will also seek the functions \( u_1(a, b, \varphi) \) and \( M_1(a, b, \varphi) \) in the form of Fourier series:
\[ u_1 = \sum_{n=-\infty}^{\infty} (p_n \cos n \varphi + q_n \sin n \varphi), \]
\[ M_1 = \sum_{n=-\infty}^{\infty} (r_n \cos n \varphi + \xi_n \sin n \varphi). \] (10)

Then, having denoted \( q_0 = (A_1/2 \sqrt{z_0}) \), \( q_1 = (B_1/2 \sqrt{z_0}) \), and \( h = (a/b) \), we obtain an infinite system of linear equations for the coefficients \( q_i \):
\[ -2 h q_0 + 2 q_1 = \frac{M_0}{z_0} \frac{G_0}{2}, \]
\[ -2 q_0 + 4 h q_1 + 6 q_2 = \frac{M_0}{z_0} \frac{G_1}{2}, \]
\[ q_1 + 6 h q_2 + 12 q_3 = \frac{M_0}{z_0} \frac{G_2}{2}, \] (11)
and for \( n \geq 3 \):
\[ (n - 1)(n - 2) q_{n-1} + 2 h (n - 1)(n + 1) q_n + (n + 1)(n + 2) q_{n+1} = \frac{M_0}{z_0} \frac{G_n}{n}. \]

Solving this system, we obtain
\[ q_0 = -\frac{M_0}{2 z_0} \frac{1}{\sqrt{z_0} \sqrt{b} \sqrt{h^2 - 1}} \left[ \frac{G_0}{2} - G_1 (h - \sqrt{b}^2 - 1) + G_2 (h - \sqrt{b}^2 - 1)^2 + \ldots + (-1)^n G_n (h - \sqrt{b}^2 - 1)^n + \ldots \right]. \] (12)