MODEL PROBLEM OF A PARTICLE IN A WAVE FIELD 
AND ITS APPLICATION TO MAGNETOElastIC 
DOMAIN WALL DYNAMICS

A. V. Kolchanov and E. A. Turov

The problem of the dynamics of a material point that interacts with some linear wave field in a finite neighborhood of the particle is formulated. It is shown that the emission of waves resulting from vibrations of the point (under the influence of the external force) can additionally lead to an acceleration of the point. The model is used to investigate the magnetoelastic dynamics of a ferromagnetic domain wall.

INTRODUCTION

It is well known that the development of general universal methods for investigating solitons (such as, for example, the inverse scattering method) has led to significant progress in their theory. However, this progress has mainly been associated with only idealized mathematical models. Study of the dynamics of solitonlike objects in the framework of any reasonably realistic model of a physical medium still encounters great difficulties and in each case requires a very "individual" approach. In fact, the only possibility for investigation remains the case in which the model contains small parameters, so that one of the forms of perturbation theory can be used.

One such method, used to describe the motion of domain walls in magnets, is the approximation of Slonczewski [1,2]. In this approximation, the shape and width of the domain wall are assumed to remain unchanged and that the entire effect of the perturbation reduces to displacement of the domain wall. Such a perturbation could be, for example, an external magnetic field. A different type of perturbation — the strain field of the elastic subsystem of the magnet — was considered in [3]. These studies proposed a simple mechanical model that permits the qualitative description of the nature of the interaction between the small "bead" fitted onto a linear elastic rod and deforming it in a certain finite neighborhood Δ of itself.

In fact, the application of this model to domain walls under conditions assumed in [3] now appears to us to have insufficient justification. However, the simplicity and transparency of the model offers the hope that it could be used to study many other systems. Therefore, its investigation was continued; in particular, several interesting nonlinear effects were found. Some results of this investigation are formulated in [4], and also in our report [5].

In the first part of this paper, we consider some generalizations of the model, and in the second we consider their application to the magnetoelastic domain wall dynamics under conditions that are formulated more clearly than in [3] and are much more stringent. In addition, we consider here the region of lower frequencies ω ≪ s/Δ (s is the wave velocity), whereas in [3] the main results applied to ω ≈ s/Δ. Other approaches to the description of magnetoelastic domain wall dynamics were used, for example, in [6,7].

A concrete result of our investigation in the second part is that we establish the possibility of instability of steady vibrations of a domain wall in a periodic external magnetic field parallel to its plane. We have found that the presence of such a field can lead to the appearance of a nonvanishing mean velocity of the domain wall associated with the reaction of asymmetrically radiated elastic waves. We note that a similar drift of a domain wall in a periodic external field can also be associated with phenomena of a quite different nature and is possible even without allowance for interaction with the elastic subsystem of the crystal — see [8,9].

1. MODELS

1.1. Lagrangian and equations of motion. We consider a simple one-dimensional model in which a material point (a particle) with mass m and coordinate R(t) interacts with some scalar wave field U(t, y). We write the Lagrange function of this system in the form
\[ L = \frac{1}{2} m \left( \frac{dR}{dt} \right)^2 + \frac{1}{2} \int_{-\infty}^{+\infty} \left[ \left( \frac{\partial U}{\partial t} \right)^2 - \left( \frac{\partial U}{\partial y} \right)^2 + l \right] dy. \]  

(1)

In (1), all quantities are assumed to be dimensionless. The wave propagation velocity is equal to unity. As function \( l \) that determines the density of the interaction, we shall use the expressions

\[ l = -BG(y - R) \frac{\partial U}{\partial y}, \]  

(2a)

\[ l = -BG(y - R) \frac{\partial U}{\partial y} \frac{dR}{dt} \]  

(2b)

or

\[ l = -BG(y - R) \frac{\partial U}{\partial y} \Lambda \left( \frac{dR}{dt} \right). \]  

(2c)

The actual form of \( l \) is determined by the functions \( G(y-R) \), and in the case (2c) \( \Lambda(dR/dt) \) also; \( B \) is a coefficient that characterizes the strength of the interaction.

As \( G \) we can consider any bounded differentiable function that decreases rapidly (at least exponentially) with growth of its argument:

\[ |G(x)| \lesssim e^{-|x|}. \]  

(3)

In addition, for definiteness we shall assume that the integral

\[ \int_{-\infty}^{+\infty} |G(x)| \, dx \]

has a value of the order of unity. The characteristic width of the region of interaction of the particle and field is also of order unity.

We consider first the Lagrangian (1) with interaction of the form (2a). To it there correspond the equations of motion

\[ \frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial y^2} = BG'(y - R), \quad m \frac{d^2 R}{dt^2} + \sigma \frac{dR}{dt} - F = B \int_{-\infty}^{+\infty} \frac{\partial U}{\partial y} G'(y - R) \, dy, \]  

(4)

where \( G'(x) \) is the derivative of \( G(x) \) with respect to its argument. In this equation, friction and the external force \( F(t) \) have been added.

1.2. Separation of the variables. We shall consider solutions of the system (4) that are defined and bounded on the infinite time interval \(-\infty < t < +\infty\). The range of variation of the coordinate \( y \) is also assumed to be infinite: \(-\infty < y < +\infty\).

We assume that waves are not incident on the particle from without. This means that at any instant \( t \) only waves traveling away from the particles can exist far from it (as \( |y| \rightarrow \infty \)). Therefore, to find \( U = U(t, y) \) we can use the retarded Green's function. Assuming that the dependence of the coordinate \( R \) on the time is known, we find from the first equation of the system (4)

\[ U(t, y) = \frac{B}{2} \int_{-\infty}^{+\infty} G[y - R(t - |x|) + z] \text{sign}(x) \, dx, \]  

(5)

where \( \text{sign}(x) = x/|x| = \pm 1 \). This expression is equivalent to the expressions with retarded potentials in electrodynamics.

Substituting the functional dependence of \( U \) on \( R \) (5) in the second equation of the system (4), we thus find an integrodifferential equation for \( R \) that is equivalent to it:

\[ m \frac{d^2 R}{dt^2} + \sigma \frac{dR}{dt} = W[R] + F, \quad W(t) = -\frac{B^2}{2} \int_{-\infty}^{+\infty} S'' [R(t) - R(t - |x|) + z] \text{sign}(x) \, dx. \]  

(6)

Here, \( W[R] = W(t) \) is an effective force that takes into account the effect of the field on the particle, and \( S'' \) is the second derivative with respect to the argument of the even convolution function

\[ S(x) = \int_{-\infty}^{+\infty} G(z) G(x + z) \, dz. \]  

(7)

Thus, to find the solution of the system (4), we must solve Eq. (6) for \( R \). After this, substituting the obtained dependence in