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ALGEBRAS OF OBSERVABLES IN THE S-MATRIX APPROACH

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The realization of algebras of observables associated with a neutral scalar field in the presence of interaction is considered. Requirements on the S matrix are formulated under which the algebras satisfy the axioms of the Haag–Araki algebraic approach.

The main aim of the present paper is, in the framework of the S-matrix approach, to construct a net of algebras of observables in the case of an interacting neutral scalar field. The fulfillment of the axioms of the Haag–Araki algebraic approach will be tested.

In accordance with the S-matrix approach, we shall assume that a quantum system can be adequately described in terms of asymptotic concepts. In particular, a pure state of a quantum system can be specified either by means of the asymptotic in configuration Φ_{in} or the asymptotic out configuration Ψ_{out}. These configurations are vectors of the Hilbert space $\mathscr{H}$ and related by the unitary operator S: $\Psi_{\text{out}}=S\Phi_{\text{in}}$.

The description of a quantum system is inseparably related to its interaction with instruments. These instruments must be regarded as external conditions (classical fields f). Clearly, the evolution of the system depends on the external conditions, and therefore the operator S must depend on f. Thus, if the system developed under "null" external conditions ($f = 0$) and had in configuration Φ_{in}, then its out configuration would be $\Psi_{\text{out}}(0)=S(0)\Phi_{\text{in}}$. If it were to develop in external conditions f and had the same in configuration Φ_{in}, then its out configuration would be different: $\Psi_{\text{out}}(f)=S(f)\Phi_{\text{in}}$. Therefore, the result of a measurement made on the system by means of the instrument f can be associated with a difference between $\Psi_{\text{out}}(f)$ and $\Psi_{\text{out}}(0)$, i.e., with a unitary operator $\mathscr{U}(f)=S^+(0)S(f)$. A connection of this type between operators and observables was pointed out in [4]. There, some properties of the algebras generated by these operators were investigated. However, these properties were not fully investigated and not sufficiently accurately. In the present paper, we analyze the algebras generated by $\mathscr{U}(f)$ in the case of a neutral scalar field.

Real measurements are made on a system in a bounded region of spacetime. Therefore, the corresponding classical fields must have compact support.

The possibility of formulating algebras of observables in terms of the operators $\mathscr{U}(f)$ can be understood on the basis of the following heuristic arguments (simultaneously, we obtain the necessary structure of the classical fields f).

We proceed from the canonical form of the S matrix:

$$S=\omega^{-1}T\exp\left\{i\int dx\mathscr{L}(x)\right\},\quad \omega=\left\langle 0\left|T\exp\left\{i\int dx\mathscr{L}(x)\right\}\right|0\right\rangle,$$

where $\mathscr{L}(x)$ is a local unrenormalized interaction Lagrangian and $|0\rangle$ is the vacuum state. Here, by the chronological product we understand the expression regularized by means of the R operation. In the case of a

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neutral field, as algebras of observables one usually understands field algebras that are constructed from Heisenberg field operators:

$$A(h)=\int dxh(x)S^{+}T(\varphi(x)S), \quad h(x)\in\mathcal{D}(R^{4}),$$

where $\varphi(x)$ is a free neutral scalar field.

However, this approach encounters great difficulties associated with the unboundedness of the $A(h)$. Note however that if we multiply by S^{+} the S matrix corresponding to a quantum-field interaction with a classical current, which is usually given by

$$S(h)=\omega^{-1}T\exp\left\{i\int dx(\mathcal{L}(x)+h(x)\varphi(x))\right\},$$

then the resulting expression can be represented as a series in chronological products of the composite fields $A(h)$. Conversely, $A(x)$ can be understood as the variational derivative of $S^{+}S(h)$ at $h=0$. It would therefore be natural to expect the algebras of observables to be generated by the operators $\mathcal{U}(h)=S^{+}S(h)$.

However, in what follows we shall see that to construct the local net of algebras of observables in the general case the algebras generated by $\mathcal{U}(h)$ are insufficiently large. Besides the Heisenberg fields $A(h)$ it is necessary to consider the interaction Lagrangian

$$L(g)=\int dxg(x)S^{+}T(\mathcal{L}(x)S), \quad g(x)\in\mathcal{D}(R^{4}).$$

Like the $A(h)$, the operator $L(g)$ is not bounded and therefore we shall construct the algebras of observables on the basis of an S matrix that depends on the classical current and the interaction switching on function:

$$S(h,g)=\omega^{-1}T\exp\left\{i\int dx[(1+g(x))\mathcal{L}(x)+h(x)\varphi(x)]\right\}. \tag{1}$$

This expression, multiplied from the left by S^{+}, can again be understood as a series in chronological products of the composite fields $A(h)$ and $L(g)$.

Since Eq. (1) has a rigorous meaning only in the framework of perturbation theory, and we do not dispose of information about the convergence of the perturbation theories, we shall not use the explicit form (1) of the operators $S(h,g)$ but extract from (1) only certain properties that we adopt as postulates.

We formulate the main assumptions.

I. The external conditions can be described by means of two real functions $h(x)$ and $g(x)$ in $\mathcal{D}(R^{4})$ (the space of test functions with compact support). We denote them by $f(x)=(h(x),\ g(x))$ and $\operatorname{supp} f\equiv\operatorname{supp} h\cup\operatorname{supp} g$, $f_1+f_2=(h_1+h_2,\ g_1+g_2)$.

We shall distinguish the space of "bare" particles and the space of physical asymptotic states ("dressed" particles). Accordingly, we adopt the following postulates.

II. Suppose $\mathcal{H}_0$ is the Fock space of "bare" particles. Then for any f there exists a unitary operator $S_0(f)$ on $\mathcal{H}_0$ which in the framework of perturbation theory has the form

$$S_0(h,g)=T\exp\left\{i\int dx(g(x)\mathcal{L}(x)+h(x)\varphi(x))\right\}.$$

III. Let $\mathcal{H}$ be the space of physical asymptotic states. Then for any f there exists a unitary operator $S(f)$ on $\mathcal{H}$ that in the framework of perturbation theory could be specified by formula (1).

In all that follows, the index $(_\alpha)$ denotes either $(_0)$ or the absence of indices; for example, $S_\alpha(f)$ is either $S_0(f)$ or $S(f)$, etc.

IV. The causality condition is satisfied in the form proposed in [4]: if $\operatorname{supp} f_1 \gtrsim \operatorname{supp} f_3$, then

$$S_\alpha(f_1+f_2+f_3)=S_\alpha(f_1+f_2)S_\alpha^{+}(f_2)S_\alpha(f_2+f_3). \tag{2}$$

The symbol $\gtrsim$ means later or spacelike. Equation (2) is a consequence of a physical requirement: systems with identical initial conditions develop in the same way if they are subject to the same conditions.

V. In $\mathcal{H}$ there is defined a unitary, strongly continuous representation $\mathcal{P}=(a,\Lambda)\to V(\mathcal{P})$ of the restricted Poincaré group, the spectral condition holding. Note that in $\mathcal{H}_0$ we have a certain representation