general set of diagonal operators (15), and it can be shown that the constants $a_1, a_2, a_3, a_4$ in (15) are the characteristics of the elliptic coordinate system itself. The construction of an orthonormal system of eigenfunctions and the determination of the eigenvalues of the complete set of diagonal operators (15), which are definite functions of the parameters $a_1, a_2, a_3, a_4$ constitute an independent problem because of certain complications encountered in its solution. Sets of harmonic functions for the special cases (16)–(20) are known or their construction does not lead to essential difficulties. Harmonic functions for the sets of operators (16) and (17) are constructed in the same way as spherocylindrical functions in $R_2$ [8]. Explicit expressions for the systems of operators (15)–(20) in the form of the corresponding differential operators are given in [11].

It should be noted that the sets of diagonal operators expressed in terms of the generators of the corresponding group enable us to give a physical interpretation to the eigenvalues of the set, i.e., to interpret the quantum numbers of the given physical problem. In this connection, it is of interest to generalize the results obtained here to the general $N$-dimensional case. The construction of all possible complete sets of observables in the space $R_{N-1}$ and the determination of the orthogonal curvilinear coordinate systems corresponding to these sets that admit separation of the variables in the corresponding Laplace equation enable one to extend significantly the manifold of harmonic functions that can be used to solve various physics problems.

LITERATURE CITED


GREEN'S FUNCTION FOR UNIAXIAL FERROMAGNET WITH
ARBITRARY SPIN AND SINGLE-ION ANISOTROPY OF SECOND ORDER

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The Green's function, low-temperature spectrum, and magnetization of an anisotropic ferromagnet with arbitrary spin and single-ion anisotropy of second order are calculated.

Introduction

The main difficulties in the calculation of the magnetization of a uniaxial Heisenberg ferromagnet consist in the presence of the anisotropy energy, whose single-ion nature prevents one using the decoupling technique characteristic of the method of two-time thermal Green's functions [1]. In this connection, various people [2–5] have shown that one can avoid decouplings in the single-ion terms, but it is then necessary to solve a system of $2S$ equations for a ferromagnet with spin $S$. It is obvious that because of the mathematical complexities this method is suitable only for small $S$.

In [6], I noted that the solution of the chain of $2S$ equations for an anisotropic ferromagnet is equivalent to the solution of a single differential equation, but for a Green's function that depends on a certain

1. Equations for the Green's Function

The Hamiltonian of the system, including exchange energy and anisotropy energy, and also the energy of the interaction of the magnetic moments with the field, has the form [2-6]

\[ H = -\frac{1}{2} \sum_{\mathbf{S}, \mathbf{S}_0} J(f \mathbf{S} \cdot \mathbf{S}_0) - D \sum_{\mathbf{S}, \mathbf{S}'_0} (\mathbf{S} \cdot \mathbf{S}')^2 - \mu B \sum_{\mathbf{S}} \mathbf{S}_0. \]  

(1.1)

The chain of equations for the two-time thermal Green's functions [1] in this case can be parametrized and reduce to the solution of a single differential equation [6]:

\[ G(x) = \frac{i}{2 \pi} \Lambda \Gamma(x), \quad \Lambda = 1 + 2 \pi a(k) G(0) \Gamma^{-1}(0), \]

\[ G(x) = \left\langle e^{i(2S, -1)S^+ S^-} \right\rangle_{\mathbf{S}, \mathbf{S}'} \quad \Gamma(x) = \left\langle (e^{i(2S, -1)S^+ S^-}) \right\rangle, \quad E_0 = a(0) + \mu B, \quad a(k) = \langle S \rangle J(k). \]

(1.2)

Equation (1.2) must be augmented by the kinematic relations for the functions \( G(x) \) and \( \Gamma(x) \), which are identical for them [7].

To establish them, we use the equation [7]

\[ \prod_{p = -\infty}^{\infty} (2S, -1 - 2p) S^+ S^- = 0. \]  

(1.3)

Using (1.3), we can show that the function \( \Gamma(x) \) satisfies the equation

\[ \prod_{p = -\infty}^{\infty} \left( \frac{d}{dx} - 2p \right) \Gamma(x) = \sum_{n=0}^{\infty} \left( \frac{d}{dx} \right)^n \prod_{p = -\infty}^{\infty} \left( \frac{d}{dx} - p \right) \Gamma(x) = 0. \]  

(1.4)

The relation (1.4) can be conveniently written in a different form by using the definition of the Bernoulli polynomials [8]:

\[ B_{2S}(x) = \sum_{n=0}^{\infty} B_{2S}(n) \frac{x^n}{n!}, \quad n = 0, 1, 2, \ldots \]

(1.5)

where \( B_{2S}(x) \) is the \( n \)-th derivative of \( \Gamma(x) \), and the polynomials \( B_{2S}(x) \) are defined by

\[ B_{2S}(x) = \frac{k!}{n!} \frac{d^{n-k}}{dx^{n-k}} \left[ (x-1)(x-2)\ldots(x-n) \right]. \]

Since Eq. (1.5) must also hold for the function \( G(x) \) (because of the symmetry of its definition to that of \( \Gamma(x) \) [7]), for it we obtain

\[ \sum_{n=0}^{\infty} \frac{1}{2^n} \left( \frac{2S}{n} \right) B_{2S-n}(S+1/2) \frac{d^n}{dx^n} G(x) = 0. \]  

(1.6)

On the basis of (1.2) and (1.6), we find

\[ \frac{d^n}{dx^n} G(x) = \left[ \frac{E-E_n}{D} \right]^n G(x) - \frac{i}{2\pi} \Lambda \frac{P_1[E-E_n, D, \phi(x)]}{P_1[E-E_n, D]}, \]

(1.7)

\[ G(x) = \frac{i}{2\pi} \Gamma(x) \Lambda \frac{P_1[E-E_n, D, \phi(x)]}{P_1[E-E_n, D]}, \]

(1.8)

\[ P_1[E-E_n, D, \phi(x)] = \sum_{n=1}^{\infty} \left( \frac{2S}{n} \right) B_{2S-n}(S+1/2)(2D)^{2S-n}(E-E_0)^{n-1} \times \sum_{m=0}^{n-1} \left[ \frac{D}{E-E_0} \right]^m \phi_m(x) = \sum_{m=0}^{n-1} C(S, m) \phi_m(x), \quad \phi_m(x) = \Gamma_m(x)/\Gamma(x), \]

(1.9)