CORRELATION OF DISPLACEMENT OF
POINT_SOURCE IMAGES

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The correlation function for displacement of the images of two point sources in an
optical system located far from the sources in a turbulent medium is considered. Exper-
imental data are compared with calculated results.

When images are transmitted over great distances atmospheric turbulence substantially affects
image quality. Distortion in the image of a point owing to a turbulent medium reduces to spreading
and displacement of the point. When the image of a line is transmitted, the spreading of individual
points leads to broadening of the line, while the displacement of the points produces either a shift in
a segment, if it is correlated, or rotation and bending of the segment if the displacements of the in-
dividual points are uncorrelated. As a consequence, in studying the transmission of an image through
a turbulent medium we must allow for and measure the correlation function for the displacement of
the image of two point sources.

The angle coordinates of the center of gravity of the random intensity distribution in the focal plane
of an optical system receiving radiation from a distant point source that has traversed a path \( L \) in a tur-
bulent medium is determined by the following expression [1]:

\[
\left\{ \begin{array}{c}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{array} \right\} = -\frac{1}{k \pi R^2} \int \int \frac{\partial S(x,y)}{\partial x} dx dy,
\]

where \( R \) is the radius of the optical system, \( S(x,y) \) is the random phase of a wave in the plane of reception,
and integration is carried out over the receiving aperture.

On the basis of (1) we obtain the following expression for the correlation of the displacement in the
centers of gravity of the images of two point sources:

\[
B_4 = \langle z_1 z_2 \rangle = \frac{1}{k^2 \pi^2 R^4} \int \int \int \int \left\langle \frac{\partial S_1(x', y')}{\partial x'} \frac{\partial S_2(x'', y'')}{\partial x''} \right\rangle \times
\]

\[
x dx' dx'' dy' dy'',
\]

or, allowing for the local homogeneity of the phase fluctuations,

\[
Fig. 1. Diagram for photographing distant point
sources: \( s_1, s_2 \) are point light sources; \( O \) is an opti-
cal system with a focal distance \( F \).
\]
The structural phase function for two point sources has been calculated in [2]. We choose the coordinate system so that the vector connecting the sources is parallel to the y axis. For locally homogeneous turbulence of the atmosphere the spectrum of the fluctuations in the dielectric constant ε can be represented as

$$\Phi_\varepsilon(\varepsilon) = 0.033 C_1^2 \varepsilon^{-13/3} \exp\left(-\frac{x^2}{\sigma_m^2}\right),$$

(5)

where \(C_1\) is the structural constant for the \(\varepsilon\) fluctuations.

Substituting the expression for \(D_S\) from [2] into (3) and taking (5) into account, we obtain

$$B_\varepsilon = \frac{0.033 C_1^2}{2 R^4} \int_{-\infty}^{\infty} K(x, y) \frac{\partial^2}{\partial x^2} \int_0^{2\pi} \int_0^{\infty} \varepsilon^{-3/2} \exp\left(-\frac{x^2}{\sigma_m^2}\right) \times
$$

$$\times \left[1 - J_0(z) \frac{x^2 + y^2}{L} \right] \left[1 + \cos \frac{x^2 z (L - z)}{kL}\right] dz dx dy,$$

(6)

where \(z = y + \sigma (L - z)/\sigma_m\), \(\sigma\) is the distance between the sources, and \(J_0\) is a zero-order Bessel function of the first kind. The inner integral with respect to \(z\) is

$$I_1 = \frac{3 \Gamma \left(\frac{1}{6}\right)}{5} \varepsilon^{-3/2} \left\{ \begin{array}{l} _1F_1\left(-\frac{5}{6}, 1, -g\right) - 1 \\ + \Re \left(1 + iD\right)^{-3/2} \left [_1F_1\left(-\frac{5}{6}, 1, -\frac{g}{1 + iD}\right) + 1 \right] \end{array} \right\},$$

(7)

where

$$g = \frac{x^2 z^2 (x^2 + y^2)}{4 L^2}, \quad D = \frac{x^2 z (L - z)}{kL},$$

(A)

and \(_1F_1(\alpha, \gamma, z)\) is a degenerate hypergeometric function. Let us evaluate the parameters \(g\) and \(D\) and their ratio \(g/D\).

Under actual radiation propagation and reception conditions (\(L = 1 \mu, \lambda_m = 2 \text{ mm}^{-1}, \sigma \approx 20 \text{ cm}, 2R = 15 \text{ cm}\) ) we can assume that \(D \gg 1\) over the entire path, except for segments several meters long at the beginning and end of the range; these will evidently not subinfluence the propagation of the waves substantially. Similar restrictions are also found when we estimate the other two parameters. Neglecting the restrictions we assume that \(g \gg 1, D \gg 1\) and \(g/D \gg 1\). Then we can use an asymptotic representation of the degenerate hypergeometric function and write \(I_1\) in the form

$$I_1 = \frac{3 \Gamma \left(\frac{1}{6}\right)}{5 \Gamma(3/6)} \left\{ \frac{12 g^{5/6}}{5 \Gamma(5/6)} - 1 - D^{5/6} \cos \frac{5 \pi}{12} \right\}.$$