An analysis is made of electromagnetic modes guided through ionospheric channels. Such fields are plotted by the method of rays at complex eikonals, and the excitation coefficients are calculated according to the Lorentz lemma. The main distinctive feature of the method based on the Lorentz lemma is that it allows for comparing various excitation mechanisms with respect to each individual mode. As specific examples we considered the excitation, by means of a small regular (local) inhomogeneity, of two ionospheric waveguides with a nonuniform distribution of parameters along the path: a plane one and a spherically symmetric one.

Specifically, the object will be to plot the fields of such modes and to determine the excitation coefficients with the sources given. A field will be plotted by the method of rays at complex eikonals and the excitation coefficients will be calculated according to the Lorentz lemma [3, 4] (also in [5, 6]). For simplicity (and thus for better clarity and more comprehensive interpretation of results), we will consider a plane isotropic model of the ionospheric channel (as in the earlier studies [1, 7, 8]). Extension to cylindrical and spherical models does not involve any fundamental difficulties [4-6, 9], as will be demonstrated at the end of this report on the example of a "whispering gallery" in a spherically symmetric model of the ionosphere.

Let us thus consider a plane waveguide channel where the dielectric permittivity \( \varepsilon(x, z) \) varies much slower in the x direction (horizontal) of mode propagation than in the transverse z direction (vertical),

\[
\left| \frac{\partial \varepsilon(x, z)}{\partial x} \right| \ll \left| \frac{\partial \varepsilon(x, z)}{\partial z} \right| \ll 1,
\]

and is independent of the y coordinate, excited by some distribution of electric currents \( j^e(x, z) \) and magnetic (in the general case) fluxes \( j^m(x, z) \). The resultant field produced by these sources outside the sources will be represented as the sum of fields due to electromagnetic modes traveling in the positive x direction, where \( x > x_s \) (\( x_s \) denoting the coordinate of the source)

\[
E_+ = \sum_I D_{+I} E_{+I}, \quad H_+ = \sum_I D_{+I} H_{+I},
\]

and due to those traveling in the negative x direction, where \( x < x_s \)

\[
E_- = \sum_I D_{-I} E_{-I}, \quad H_- = \sum_I D_{-I} H_{-I}.
\]

Here \( D_{+I} \) are mode excitation coefficients.

With regard to the polarization characteristics, one must distinguish between waves of the transverse electric (TE or H) kind whose components are

\[
E_y = U(x, z), \quad H_x = -\frac{i}{k_0 Z_0} \frac{dE_y}{dz}, \quad H_z = \frac{i}{k_0 Z_0} \frac{dE_x}{dx}
\]

and those of the transverse magnetic (TM or E) kind whose components are

\[
H_y = V(x, z), \quad E_x = \frac{iZ_0}{k_0 \varepsilon(x, z)} \frac{dH_y}{dx}, \quad E_z = -\frac{iZ_0}{k_0 \varepsilon(x, z)} \frac{dH_z}{dz},
\]

where \( k_0 = \omega \sqrt{\varepsilon_0 \mu_0}, \ Z_0 = \sqrt{\mu_0 / \varepsilon_0}, \ \varepsilon_0 \) is the dielectric permittivity of vacuum and \( \mu_0 \) is the magnetic permeability of vacuum (in the practical rational system of units, with the time factor \( e^{i\omega t} \)). Functions \( U(x, y) \) and \( V(x, y) \)

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satisfy the equations

\[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} + k_0^2 \varepsilon(x, z) U = 0; \quad (4) \]

\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} - \frac{\partial \varepsilon}{\varepsilon} \frac{\partial V}{\partial x} - \frac{\partial \varepsilon}{\partial z} \frac{\partial V}{\partial z} + k_0^2 \varepsilon(x, z) V = 0. \quad (4') \]

We will henceforth assume everywhere that \( \varepsilon(x, z) \) varies rather slowly to the scale of the local wavelength, i.e., that the conditions

\[ \left| \frac{1}{k_0^2 \varepsilon} \frac{d \varepsilon}{dz} \right|^2 \ll 1, \quad \left| \frac{1}{k_0^2 \varepsilon} \frac{d^2 \varepsilon}{dz^2} \right| \ll 1 \quad (5) \]

are satisfied. Within the validity range of these conditions and condition (1), as can be easily ascertained, Eq. (4') reduces to Eq. (4) upon the substitution \( V(x, z) = \sqrt{\varepsilon(x, z)} V_H(x, z) \).

The solution to Eq. (4) will be sought in the form

\[ U_{\pm l}(x, z) = \frac{1}{V_{\pm l}(x)} \exp \left\{ \mp i \int h_{\pm l}(x) dx \right\}, \quad (6) \]

where \( h_{\pm l}(x) \) is the mode propagation constant. Inserting expression (6) into Eq. (4) yields

\[ \frac{\partial^2 U_{\pm l}}{\partial z^2} - \frac{1}{2} \frac{\partial h_{\pm l}}{\partial x} \left( \frac{1}{h_{\pm l}^2} \frac{\partial h_{\pm l}}{\partial x} \right) U_{\pm l} - \frac{1}{2} \frac{\partial U_{\pm l}}{\partial x} + \frac{1}{h_{\pm l}^2} \left( \frac{1}{3} \left( \frac{1}{h_{\pm l}^2} \frac{\partial h_{\pm l}}{\partial x} \right)^2 \right) U_{\pm l} = 0. \quad (7) \]

The expressions for \( U_{\pm l}(x, z) \), which are asymptotic with respect to the small parameter \( \varepsilon_{\pm l} = \frac{1}{k_0^2 \varepsilon} \frac{d \varepsilon}{dz} \), are valid, of course, only sufficiently far from the reversal point \( \kappa_{\pm l}(z_0) = 0 \), * can be obtained by applying the method of rays at complex eikonals (this method has been described in detail in [2]). † With an accuracy down to terms of third-order smallness (such an accuracy is, according to the estimates made in [1], entirely adequate under conditions of the earth's ionosphere), we obtain

\[ U_{\pm l} = \left\{ \begin{array}{l} \frac{1}{V_{\pm l}(x)} \exp \left\{ \pm i \int h_{\pm l}(x) dx \right\} \left[ C_{1<} \exp(-i \varphi_{<}(x, z)) + C_{2<} \exp(i \varphi_{<}(x, z)) \right] \quad (z < z_0) \\
\frac{1}{V_{\pm l}(x)} \exp \left\{ \pm i \int h_{\pm l}(x) dx \right\} \left[ C_{1>} \exp(-i \varphi_{>}(x, z)) + C_{2>} \exp(i \varphi_{>}(x, z)) \right] \quad (z > z_0) \end{array} \right. \quad (8) \]

where \( \kappa_{\pm l} = \sqrt{h_{\pm l}^2 - k_0^2} = -1, C_{1<}, C_{2<}, C_{1>,} C_{2>} \) are constants,

\[ \varphi_{<}(x, z) = \int_{z_0}^z \frac{h_{\pm l}(x) a_{\pm l}^2}{2x_{\pm l}(x, z)} + \frac{h_{\pm l}(x) a_{\pm l}}{x_{\pm l}(x, z)} \, dz; \]

\[ \varphi_{>}(x, z_0, z) = \int_{z_0}^z \frac{h_{\pm l}(x) a_{\pm l}^2}{2x_{\pm l}(x, z)} + \frac{h_{\pm l}(x) a_{\pm l}}{x_{\pm l}(x, z)} \, dz. \]

* In [1] \( \alpha_{\pm l} \) called the adiabaticity parameter.
† These expressions are valid, of course, only sufficiently far from the reversal point. In the immediate vicinity of the reversal point we must resort to a more precise analysis, which evidently is also possible by this method with the aid of corresponding calibration functions.