SUPERRADIATION IN ENSEMBLES
OF CLASSICAL ELECTRON OSCILLATORS
UNDER CONDITIONS OF GROUP SYNCHRONISM

N. S. Ginzburg, I. V. Zotova,
and A. S. Sergeev

We consider the cyclotron superradiation of a moving electron bunch under conditions of group synchronism when the forward velocity of the bunch is equal to the group velocity of the electromagnetic wave. It is shown that this mode of operation makes it possible to increase the increment of superradiant instability and raise the peak radiated power.

1. At present, considerable attention is drawn to investigating induced radiation in spatially localized excited electron ensembles with an infinite (in the radiation time scale) lifetime of electrons. It is shown [1-4] that the radiation "self-organization" effects are important under such conditions. These effects result in electron phasing followed by coherent irradiation of stored energy in the form of short quasimonochromatic pulses. By analogy with quantum electronics, this process was named the superradiation (SR).

In the present paper, we analyze the specific features of cyclotron SR under conditions of group synchronism when the forward velocity of an electron bunch coincides with the group velocity of the electromagnetic wave, i.e.,

\[ V_\parallel = V_{gr}. \]  

A similar situation occurs, for example, in waveguiding of radiation (or in a plasma) when the dispersion curves of the wave \( h = c^{-1}\sqrt{\omega^2 - \omega_c^2} \) and of the electron flow \( \omega - hv_\parallel = \omega_H \) are tangential to each other (Fig. 1a). In this case, the following relations are satisfied:

\[ \omega_H = \omega_c \gamma_\parallel^{-1}, \quad \omega = \gamma_\parallel^2 \omega_H, \]  

where \( \omega_c \) is the cutoff frequency, \( \omega_H = eH_0/mc\gamma \) is the relativistic gyrofrequency,

\[ \gamma_\parallel = (1 - V_\parallel^2/c^2)^{-1/2}, \quad \gamma = (1 - V_\parallel^2/c^2 - V_\perp^2/c^2)^{-1/2}, \quad V_\perp = \beta_\perp c \]

is the rotational velocity of electrons. Hence, the radiation frequency \( \omega \) of ultrarelativistic electrons may considerably exceed their oscillation frequency. Moreover, it will be shown below that the group synchronism condition (1) is favorable from the point of view of raising peak power and superradiant instability increments.

2. Further analysis was conducted in a tracking reference frame \( K' \) moving with the translational velocity of the electron bunch. In this system, the longitudinal wavenumber \( h' \) and the transverse component of the magnetic field \( H'_\perp \) tend to zero and the problem reduces to investigating the radiation of an immobile ensemble of cyclotron oscillators at a quasicritical frequency (Fig. 1b). Choosing \( \omega_c \) as the carrier frequency and assuming that the transverse structure of the radiation coincides with the structure of the waveguide...
mode $\vec{E}_1'(r'_\perp)$, we represent the field acting on electrons in the form

$$\vec{E}' = \text{Re}[\vec{E}_1'(r'_\perp)A'(z',t')\exp(i\omega_c t')]$$.

Then, in the case of arbitrary relativism, the superradiation process can be described by the following system of equations:

$$i\frac{\partial^2 a}{\partial Z^2} + \frac{\partial a}{\partial \tau} = 2if(Z)GJ, \quad J = \frac{1}{\pi} \int_0^{2\pi} \frac{\hat{p}_+}{\sqrt{1 - |\hat{p}_+|^2}} d\Theta_0. \quad (4)$$

$$\frac{\partial \hat{p}_+}{\partial \tau} + i\hat{p}_+(1 - \Delta - \frac{1}{\sqrt{1 + |\hat{p}_+|^2}}) = ia. \quad (5)$$

$$\hat{p}_+|_{\tau=0} = \gamma'_10\beta_0'\exp(i\Theta_0), \quad \Theta_0 \in [0, 2\pi], \quad a_\tau=0 = a_0(Z).$$

The dimensionless quantities are denoted as follows:

$$\hat{p}_+ = (p'_x + ip'_y)/mc$$

is the normalized transverse momentum of electrons,

$$a = \frac{eA'_1}{mc^2\omega_c} \cdot J_{m-1}(R_0\omega_c/c), \quad Z = z'\sqrt{2}\omega_c/c, \quad \tau = t'\omega_c,$$

$$\Delta = (\omega'_H - \omega_c)/\omega_c$$ is the detuning of unperturbed cyclotron frequency from cutoff frequency,

$$\frac{eI_0}{mc^3} \cdot \frac{1}{\beta^4_{10}\beta_{10}||\gamma||} \cdot \frac{\lambda^2}{\pi R^2} \cdot \frac{J^2_{m-1}(R_0\omega_c/c)}{J^2_{m}(\nu_n)(1 - m^2/\nu_n^2)}$$

is the form factor written under the assumption that the electron bunch has a tubular configuration with the injection radius $R_0$, $I_0$ is the total current in the laboratory reference frame, $\lambda = 2\pi c/\omega_c = 2\pi R/\nu_n$, $R$ is the radius of the waveguide, $m$ is the azimuthal index of the waveguide mode, and $\nu_n$ is the $n$th root of the equation $J_m(\nu) = 0$. The function $f(Z)$ describes the longitudinal distribution of electron density. In the case of a thin layer $b'^2/c\lambda T' \ll 1$, where $T'$ is the characteristic time of interaction (inverse increment), we can assume $f(Z) = = \sqrt{2}(\omega_c/c)b'(Z)$. Assuming that the signal is weak, representing the radiated field in the form $a(Z, \tau) = a(0)\exp(-i\hbar|Z| + i\Delta \tau + i\Omega \tau)$ and linearizing the system of equations (4), we arrive at the characteristic equation ($G = GB^2\gamma_0^{-2}$)

$$i\Omega^2\sqrt{\Omega^2 + 2\dot{G}\Omega} = 2\dot{G},$$

which determines the complex eigenmode frequencies of the layer.