FLUCTUATIONS IN THE INTENSITY AND FREQUENCY OF A LASER

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A study is made of fluctuations in the field of a single mode continuous laser. Calculations are carried out on the basis of phenomenological parameters. Formulas are obtained for the spectral densities of the intensity and frequency fluctuations. Some of the results are compared with existing experimental data.

The present paper is concerned with calculating the fluctuations in the field of a continuous laser. In previous papers on this topic (see [1, 2], for example), insufficient attention has been given to the dispersive and nonlinear properties of the active medium. Also, no formulas have been obtained, to date, for the spectra of the fluctuations. Our analysis is based on the phenomenological description of the properties of the active medium, which is characterized by its electric polarization (the results are readily carried over to the case of magnetic polarization). Starting from simple physical considerations, this approach provides a quite general solution to the problem and yields expressions for the spectral densities of the fluctuations and for the natural linewidth of the radiation. The numerical values of the parameters occurring in these formulas may be worked out on the basis of some concrete model or other, or determined experimentally.*

The analysis is carried out for the case of a laser oscillating under single-mode conditions; some qualitative considerations are given at the end of the paper for the case of multimode operation.

The low-frequency region of the spectrum of the fluctuations is of greatest interest. In this region it is permissible to replace the differential equations coupling the polarization of the medium and the field (and other parameters) by a direct (algebraic) relationship. This approach is valid to frequencies of the order of $1/\tau$, where $\tau$ is the relaxation time of the medium. In gas lasers $\tau \approx 10^{-7} - 10^{-8}$ sec, and consequently such calculations may be regarded as valid up to frequencies of the order of $10^7$ cps.**

When formulating the equations for the electric and magnetic fields $E$ and $H$, we shall idealize the problem in the normal manner, and assume that the field is purely transverse and varies only along the $x$ axis of the resonator. We shall assume that the active medium is uniform and that its polarization is given by the relationship $P = \kappa E = (\kappa_1 - i\kappa_2)E$, where $\kappa_1$ and $\kappa_2$ are functions of the intensity and frequency of the oscillations but not of time. We shall allow for the losses in the resonator (diffraction losses on reflection from the mirrors, and so on) by introducing a uniformly distributed ohmic conductivity $\sigma$ (the introduction of boundary conditions is thereby avoided, which helps to keep the analysis from becoming too complicated). The numerical value of $\sigma$ may be found from the experimentally measured bandwidth of the resonator.

The parameters $\kappa_2$ and $\sigma$ characterizing the energy dissipation (positive or negative) enable the fluctuations in the system to be worked out by introducing parasitic random fields delta-correlated in space (this question is discussed in detail in [5] for the case of equilibrium radiation). We shall adopt the completely equivalent procedure of introducing parasitic random components into the polarization and current density, denoted respectively by $\rho$ and $j$.

On the basis of the foregoing remarks, we write for $E$ and $H$ in the resonator

$$\frac{\partial E}{\partial t} = - \frac{1}{c} \frac{\partial H}{\partial x},$$

$$\frac{\partial H}{\partial x} = - \frac{1}{c} \frac{\partial E}{\partial t} - \frac{4\pi}{c} \frac{\partial (P + \rho)}{\partial t} - \frac{4\pi}{c} (\sigma E + j).$$

The spectral densities of $\rho$ and $j$ are readily worked out in the case when no oscillations are present in the system. We consider a small volume in the resonator $dV = Sdx$, where $S$ is the cross sectional area of the resonator, and apply to it the well-known Callen-Welpton relationship, whence we obtain for the spectral densities of $j$ and $\rho$

$$\rho^2_i = \frac{2}{\pi} \frac{\sigma}{Sdx} \theta_i, \quad j^2_i = \frac{2}{\pi} \frac{\kappa_i}{Sdx} \theta_i,$$

where

$$\theta_i = \frac{\hbar \omega}{2} + \frac{\hbar \omega}{\exp (\hbar \omega / kT) - 1} \quad (i = 1, 2).$$

The quantity $\sigma$ characterizes the dissipation of energy from the resonator into the surrounding space, and it is natural to regard $T_i$ in the expression for $\theta_i$ as the temperature of this space. Consequently, the quantity $\rho^2_i$ is unaffected by the presence or absence of oscillations in the system. The significance of $\rho^2_i$ when oscillations are present is a more complicated matter, however, to which we shall return later.

*Alternatively, an experimental study of the fluctuations may determine some parameters (properties) of the system which would be difficult or impossible to determine from its dynamic behavior.

Assuming that the system is almost conservative, we shall seek the solution of Eqs. (1) and (2) as an expansion in terms of the eigenfunctions of the corresponding conservative system. Putting

\[ E = e \sin (Kx), \quad H = h \cos (Kx), \]

we obtain equations for \( e \) and \( h \).*

\[ \frac{dh}{dt} = cKe, \]

\[ \frac{de}{dt} = cKh - \frac{8 \pi}{L} \frac{de}{dt} \int_0^L \sin^2 (Kx) dx, \quad \xi = -4\pi e - \xi, \]

where

\[ \xi = \frac{8 \pi}{L} \int_0^L \sin (Kx) dx \]

and

\[ \eta = \frac{8 \pi}{L} \int_0^L \rho \sin (Kx) dx \]

are random functions of time, \( L \) is the resonator length, and \( x_2 - x_1 = l \) is the length of the active medium.

Let us determine the spectral densities of \( \xi \) and \( \eta \) in the absence of oscillations in the system. In this case \( \xi \) and \( \eta \) do not depend on \( x \), and utilizing (3) and (4) we obtain

\[ \bar{\xi}^2 = \frac{64 \pi^2}{L^2} \int_0^L (\frac{\bar{\xi}}{\bar{\rho}} dx) \sin^2 (Kx) dx = \frac{64 \pi^2}{V} \xi, \]

\[ \bar{\eta}^2 = \frac{64 \pi^2}{L^2} \int_0^L (\frac{\bar{\eta}}{\bar{\rho}} dx) \sin^2 (Kx) dx = \frac{64 \pi^2}{V} \eta, \]

where \( V = L S \) is the resonator volume.**

The validity of the above considerations may be checked by putting \( \xi = \eta = 0 \) (the equilibrium case). The spectral density \( \bar{\xi}^2 \) may be found from (7) and (8), and then the mean square \( \bar{\xi}^2 = \int_0^\infty \bar{\xi}^2 d\omega \). The mean energy in the resonator may be readily found by integrating over the entire resonator volume; it turns out to equal \( \bar{\theta} \), as indeed it must do in this case.

Spontaneous oscillations are possible in the system only provided \( \xi < 0 \) and provided \( \eta \) is sufficiently large. Under conditions of steady-state oscillation \( \eta \) will be a function of \( x \) (it is natural to regard \( \eta \) as de-

*We note that \( K \) is not a constant along the entire length of the resonator. It is not necessary to take this into account, however, as far as our present purposes are concerned. Reflections from boundaries within the resonator are neglected.

**Results (11) and (12) may, of course, also be obtained by introducing the delta-correlated (in space) quantities \( j \) and \( \rho \).