A numerical method is described for calculating the coefficient of reflection from a dielectric wedge formed in a rectangular waveguide by the oblique interface between media having different permittivities. The plane of the interface is perpendicular to the broad walls of the waveguide. Results are reported for calculations and for an experimental check in the 3-cm band for specific wedge configurations.

1. INTRODUCTION

The reflections of electromagnetic waves from a dielectric wedge formed by the interface between media having differing permittivities are of interest in the matching of microwave devices making use of dielectric materials.

The multimode approximation was used in [1, 2] to consider reflection from an oblique dielectric plane in a rectangular waveguide with the plane perpendicular to the broad wall. If a single mode propagates in the waveguide with the dielectric, however, as is typical of most microwave devices, the single-mode approximation can provide adequate accuracy for practical purposes. Such an approximation has proved extremely effective in analogous problems (a plane separating dielectrics perpendicular to narrow walls of a rectangular waveguide) [3, 4]. Moreover, there is a significant reduction in the amount of computation.

2. CALCULATION METHOD

Let a dielectric wedge formed by media differing in permittivity have a length $L$ along the $z$ axis (Fig. 1). Assuming a wave close in structure to a TE$_{10}$ mode propagates along a segment of a rectangular waveguide having a longitudinally inhomogeneous filler, we may follow [5] and write the equation for the distribution function of the transverse electric field as

$$\frac{d^2 \Psi}{dz^2} + \beta^2(z) \Psi = 0,$$

where $\beta(z)$ is the local, $z$-dependent propagation constant.

Making use of the results of [6, 7], after uncomplicated manipulations, we may represent the local propagation constant $\beta(z)$ for the given wave in the following form:

For a nonsymmetric interface (Fig. 1a):

$$\beta^2(z) = k_0^2 \left[ \varepsilon_1 + (\varepsilon_z - \varepsilon_1) \left( \frac{z}{L} - \frac{1}{2} \sin \frac{2\pi z}{A} \right) \right] \frac{\pi^2}{a^2},$$

For a symmetric interface (Fig. 1b):

$$\beta^2(z) = k_0^2 \left[ \varepsilon_1 + (\varepsilon_z - \varepsilon_1) \left( \frac{z}{L} + \frac{1}{2} \sin \frac{\pi z}{L} \right) \right] \frac{\pi^2}{a^2},$$

For a symmetric interface (Fig. 1c):

$$\beta^2(z) = k_0^2 \left[ \varepsilon_1 + (\varepsilon_z - \varepsilon_1) \left( \frac{z}{L} - \frac{1}{2} \sin \frac{\pi z}{L} \right) \right] \frac{\pi^2}{a^2}.$$
where \( \varepsilon_1, \varepsilon_2 \) are the relative permittivities of the media in waveguides having a homogeneous filler; \( a \) is the waveguide width; \( L \) is the length of the wedge; \( k_0 = 2\pi / \lambda \) is the wave number of free space. Taking (2)–(4) into account and introducing the new variable \( \xi = z / L \), we write (1) as

\[
\frac{d^2 \Psi}{d \xi^2} + (\alpha + 2\lambda + \gamma \sin \theta) \Psi = 0,
\]

where

\[
\alpha = \left( \frac{\pi L}{\lambda} \right)^2 \left[ 4\varepsilon_1 - \left( \frac{\lambda}{\theta} \right)^2 \right], \quad \beta = (\varepsilon_2 - \varepsilon_1) \left( \frac{2\pi L}{\lambda} \right)^2,
\]

\[
\gamma = \frac{\varepsilon_2 - \varepsilon_1}{2\pi} \left( \frac{2\pi L}{\lambda} \right)^2 \quad (q = 2\pi) \quad (\text{Fig. 1a}),
\]

\[
\gamma = \frac{\varepsilon_2 - \varepsilon_1}{\pi} \left( \frac{2\pi L}{\lambda} \right)^2 \quad (q = \pi) \quad (\text{Fig. 1b}),
\]

\[
\gamma = -\frac{\varepsilon_2 - \varepsilon_1}{\pi} \left( \frac{2\pi L}{\lambda} \right)^2 \quad (q = \pi) \quad (\text{Fig. 1c}).
\]

The two linearly independent solutions of (5) must satisfy the following conditions: \( \Psi_1(0) = 1, \Psi_1'(0) = 0, \Psi_2 = 0, \Psi_2'(0) = 1 \).

From the condition for continuity of \( \Psi \) and \( \Psi' \) at the ends of a waveguide having an inhomogeneous sphere we obtain an expression for the reflection coefficient,

\[
\Gamma = \frac{-\Psi_1' - \beta_1 \Psi_2 + j(\beta_2 \Psi_1' - \beta_1 \Psi_1)}{\Psi_1' + \beta_1 \Psi_2 + j(\beta_2 \Psi_1' + \beta_1 \Psi_1)},
\]

where \( \beta_1, \beta_2 \) are the respective propagation constants for homogeneous unfilled and filled waveguides.

The values of the functions \( \Psi_{1,2} \) and \( \Psi_{1,2}' \) occurring in (4) are calculated at the point \( \xi = 1 \) corresponding to the end of the segment with inhomogeneous filler.

3. RESULTS OF NUMERICAL CALCULATIONS

We make use of the following scheme to compute \( \Psi_{1,2} \) and \( \Psi_{1,2}' \). Making the substitution \( d\Psi / d\xi = \varphi \), we transform (5) to a system of two first-order differential equations with corresponding initial conditions. The latter is solved by the standard Runge–Kutta method.

As an illustration, Fig. 2 shows solutions of (5) and their first derivatives at the point \( \xi = 1 \) as functions of the angle of inclination \( \theta \) for \( \varepsilon_2 = 2, \varepsilon_1 = 1, a / \lambda = 0.6 \) corresponding to the structure of Fig. 1a. As we see,