CONVERGENT INSTABILITY IN THE IONOSPHERE

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A linear theory of the convergent instability (CI) of ionospheric plasma associated with the nonuniform nature of its regular motion is examined. The conditions under which CI appears in the E- and F-layers for vertical ion motion caused by various physical factors are analyzed. The possibility of small-scale strongly geomagnetic-field-aligned nonuniformities of electron concentration ($l_{\text{min}} \sim 10-30 \text{ m}$) is demonstrated. The altitude dependence of collision frequency is shown to play a large role in CI.

The question of convergent instability (CI) due to nonuniform regular charged-particle motion in ionospheric plasma was first raised by Gershman [1], who found that for weakly aligned nonuniformities $\gamma \propto \text{div}U_{\text{i}0}$ (where $\gamma$ is the CI increment and $U_{\text{i}0}$ is the regular ion velocity). It was found later that $\gamma \propto \text{div}U_{\text{i}0}$ for strongly aligned nonuniformities [2]. A formula for an arbitrary alignment in this approach was provided by Gel'berg [3]. Subsequently, CI was practically ignored, although the role of the term $\partial U_{\text{i}0}/\partial z$ in nonuniformity formation in internal-wave propagation was analyzed using another approach [4]. Here we shall derive a complete formula for the CI increment on the basis of a procedure proposed earlier [5] and analyze the role of that instability in the formation of nonuniform E- and F-layer structures. Preliminary results were published elsewhere [6].

Convergent instability is analyzed on the basis of quasi-hydrodynamic equations for ions and electrons [4, 7] using a conventional method for study of the instabilities of ionospheric plasma [7]. Following the procedure [5], however, after linearizing the initial equations, we replace the perturbations of the charged-particle concentrations $N_1$ by a new variable $n_1 = N_1/N_0$ (all unperturbed and perturbed quantities receive subscripts "0" and "1," respectively; it is assumed that the plasma is quasi-neutral, so that the unperturbed and perturbed ion and electron concentrations are equal: $N_{\text{io}} = N_{\text{e0}} = N_0$ and $N_{\text{i1}} = N_{\text{e1}} = N_1$). Then, after simplifications [7], the linearized equations in a linear approximation have the form

$$\frac{\partial n_1}{\partial t} + n_1(\text{div}U_{\text{a0}} + \vec{K}_0\cdot \vec{U}_{\text{a0}}) + \vec{U}_{\text{a0}} \cdot n_1 + \text{div}\vec{U}_{\text{a1}} + \vec{K}_0\cdot \vec{U}_{\text{a1}} = 0$$

(1)

where $e_\alpha$, $m_\alpha$, $w_{\text{aH}}$, $v_{\text{aH}}$, $U_{\text{a0}}$, $U_{\text{a1}}$, and $V_{T\alpha}$ are the charge, mass, gyrofrequency ($w_{\text{aH}} = e_\alpha H_0/m_\alpha c$), the frequency of collisions with other particles ($v_{\text{aH}} = v_{\text{an}} + v_{\text{eH}}m_\alpha/e_\alpha$), where $v_{\text{an}}$ is the frequency of collisions with neutral particles; in this approach, allowance for the Coulomb collision frequency $v_{\text{eH}}$, while not exact, is usually entirely sufficient [8]), and the regular, perturbed, and thermal velocities ($V_{T\alpha}^2 = xT_\alpha/m_\alpha$, $T_\alpha$ is temperature, and $x$ is Boltzmann's constant) of particles of type $\alpha$. For electrons, $\alpha = e$; for ions, $\alpha = i$. $H_0$ is the geomagnetic-field strength, $c$ is the velocity of light in vacuum, $H_0 = H_0/H_0$, and $K_0 = H_0/H_0$: $K_0 = \text{VNG}_0/N_0$. The electric-field perturbation is assumed to be potential $\vec{E}_1 = -\nabla\Phi$.

The introduction of variable $n_1$ is advisable in that the coordinate dependence of the coefficients of differential Eqs. (1), which is associated with a corresponding dependence of $N_0$, remains only in the continuity equation and only in terms of $K_0$, which has a weaker coordinate dependence than does $N_0$. In particular, $K_0$ is entirely coordinate-independent for an exponential approximation $N_0 \propto \exp(-z/L)$. This improves the conditions for use of a quasi-local approach, which involves the representation of all perturbed quantities in the form of a Fourier expansion.
whose amplitudes $f_{k,w}$ vary so slowly that their derivatives are negligible. Here $w$ is the complex frequency ($w = w_d - i\gamma$) and $\vec{k}$ is the wave vector ($k = 2\pi/l$, where $l$ is the scale of the nonuniformity). In the presence of a coordinate dependence of $K_o$, a quasi-local approximation is valid for $k > > K_o$. The need to introduce $n_1$ is supported by the physical significance of the results obtained in this case. Firstly, now $N_1 \propto N_0$, which has been confirmed by numerous experiments [8]; secondly, the description of convergent instability, which is to be discussed below, becomes complete and physically comprehensible.

After substitution of expansions (2) into Eqs. (1), assuming the existence of a nontrivial solution, we obtain a dispersion equation, which for the CI increment gives

$$\gamma = -N_0^{-1}\text{div}(N_0\vec{U}_{i0}) - k^2 D_{at},$$

(3)

where $D_{at} = [d_{eo}/(1 + \Psi_\theta)](m_{ei}/m_{ei}e)eD_a$ is the coefficient of ambipolar diffusion in the presence of a magnetic field, $D_a = V_e^2/\rho_e(1 + T_e/T_i)$, $\Psi_\theta = (m_{ei}/m_{ei}e)(d_{eo}/d_{io})(1 + \beta^2)$, $d_{at} = \beta_a + \cos^2 \theta$ ($\theta$ is the angle between $\vec{k}$ and $\vec{B}_o$), and $\beta_a = v^2_k/\omega_0$. Formula (3) was derived under the following condition of quasi-neutrality:

$$\text{div}(N_0\vec{U}_{e0}) = \text{div}(N_0\vec{U}_{i0}).$$

(4)

The formula for $w_0$, ignoring the term proportional to $D_{at}$, coincides with a formula obtained by Gershman et al. [8].

The CI increment is determined not by velocity divergence, as had been assumed earlier [1-4], but by flux divergence, which is more correct from a physical point of view. This results in an additional term of the form $K_0\vec{U}_{i0}$, which describes perturbation transport along the regular electron-concentration gradient, which is equivalent to the relative intensification of the perturbations. Thus, the instability is purely convective. Here it is important that $\vec{U}_{i0}$ be a stationary velocity, which is of the factors that enters into the formation of the $N_0$ profile itself. But in this case the profile is not carried away as a whole at this velocity, although the plasma is "driven" through it, transporting the nonuniformities relative to the regular concentration gradient. Now the "old" term $\gamma \approx \text{div} \vec{U}_{i0}$, regardless of the degree of alignment of the nonuniformities to the regular concentration gradient. Now the "old" term $\gamma \approx \text{div} \vec{U}_{i0}$, regardless of the degree of alignment of the nonuniformities to the regular concentration gradient.

For CI analysis it is necessary to determine $\vec{U}_{i0}$, which involves difficulties due to the interconnected motion of the electron and ion components in an inhomogeneous plasma through the resulting internal electric field $\vec{E}_{in}$. We shall proceed as follows to solve this problem. We shall assume that all quantities are dependent only on altitude $z$, since this is the dominant dependence in the overwhelming majority of cases in ionospheric plasma. Then Eq. (4), which now has the form

$$\frac{\partial}{\partial z}\left(N_0U_{i0z}\right) = \frac{\partial}{\partial z}\left(N_0U_{e0z}\right),$$

(5)

can be integrated with respect to $z$ and used to find $\vec{E}_{in}$, which has only a $z$-component in this case. If $N_0U_{i0z}$ and $N_0U_{e0z}$ are smooth functions of $z$ that decrease to zero at infinity, the resulting constant of integration $C$ can be made equal to zero. This is obvious, since otherwise there would be instability ($\gamma \neq 0$) even in the absence of velocities $\vec{U}_{i0z}$ (see below), for which there is no basis. This is indicated by the fact that for the sporadic E-layer, results are obtained below that coincide with results obtained using an entirely different approach [4, 7].

There are also physical bases for letting $C = 0$. Firstly, the resulting equation $U_{i0z} = U_{e0z}$ has a transparent physical interpretation, involving the essence of CI itself, which is that of transport of the nonuniformity as a whole with preservation of neutrality. Therefore, if all parameters are dependent on $z$ alone, the vertical velocity components must be equal. Otherwise, there would be charge separation and loss of neutrality. In fact, this means that to ensure neutrality the internal electric field alone must equalize the vertical ion and electron velocities. Secondly, if all but diffusional plasma motion is removed, the equality $U_{i0z} = U_{e0z}$ is satisfied by the ambipolar nature of the diffusion itself, so that $C = 0$.

The particle velocities are determined by the formula [4, 7]:

$$\vec{U}_{a0} = (1 + \beta_\alpha^2)^{-1}\{\beta_\alpha^2\vec{V}_{a1} + \beta_\alpha\vec{V}_{a1} \cdot \vec{h}_0\} + \vec{h}_0(\vec{V}_{a1}, \vec{h}_0).$$