NATURAL WAVES OF A PERIODIC ARRAY OF RECTANGULAR
LONGITUDINALLY MAGNETIZED FERRITE RODS*

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Electromagnetic waves of a two-dimensionally periodic array of parallel
rectangular longitudinally magnetized ferrite rods are investigated. It is
assumed that all the rods are identical and are located at the nodes of an
oblique-angled grid. The solution of the boundary-value problem is constructed
by Galerkin's method. The convergence of the solution is investigated
numerically, and the dependences of the propagation coefficients of the lowest
waves on geometrical parameters of the rod array and the characteristics of the
gyromagnetic medium are calculated.

It is of interest in connection with the use of open gyromagnetic structures for the
construction of radiating systems of multielement phased antenna arrays to use the charac-
teristics of natural waves of a system of gyromagnetic rods with the coupling among them
taken into account.

The method of partial regions is effective for the analysis of arrays of circular rods. Thus,
an array of circular laminated longitudinally magnetized rods has been discussed in
[1] by this method in the treatment of [2]. A periodic array containing several circular
rods in a cell which differ in the gyromagnetic medium parameters has been investigated in
[3] by the procedure developed in [2] and [4].

The use of the projection method [5] makes it possible to reduce the problem of natural
waves of an array of gyromagnetic rods to a well-known algebraic form [6] for an arbitrary
transverse cross-sectional shape of the rods. The authors of [7] have communicated about
the development by this same method of an algorithm for finding with the Floch boundary
conditions the natural waves of a rectangular waveguide which is filled with a magnetized
ferrite.

The development of the projection algorithm proposed in [8] for the analysis of an
array of rectangular dielectric rods is given in this paper for a two-dimensionally periodic
array of rectangular longitudinally magnetized rods.

Formulation and Algebraization of the Problem. The structure being investigated, which
is uniform along the Oz axis and periodic in the transverse plane, is shown in Fig. 1. It
has been assumed that the ferrite rods are identical, uniformly magnetized along the Oz
axis, and surrounded by an isotropic medium with permeabilities \( \varepsilon_0 \) and \( \mu_0 \). The ferrite
medium is described by the dielectric constant \( \varepsilon_1 \) and the magnetic permeability tensor

\[
\mu = \begin{pmatrix}
\varepsilon_1 & -j\mu_2 & 0 \\
\mu_2 & \varepsilon_1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

The investigation of natural electromagnetic waves in the structure under discussion
reduces to the solution of the equation

\[
L_H H_\perp = p^2 k^2 H_\perp,
\]

where \( p = \beta/k \) is the propagation coefficient of the wave; \( \beta \), longitudinal wave number;
\( k = \omega / \sqrt{\varepsilon_0 \mu_0} \), \( \omega \), harmonic vibration frequency; and \( H_\perp \), transverse intensity vector of the

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magnetic field, which satisfies the Floch quasiperiodicity condition:

\[ H_0(x_1 + d_1, y_1 + d_2) = H_0(x_1, y_1) \exp \left[ -j(\psi_1 + \psi_2) \right] \]  

(1)

where \( d_1 < x_1 < d_1 \), \( d_2 < y_1 < d_2 \), and \( \psi_1 \) and \( \psi_2 \) are the phase advances of the field on periods of the structure specified in the intervals \( \pi < \psi_1 < \pi \) and \( -\pi < \psi_2 < \pi \).

The elements of the differential operator \( L^H_1 \) are defined by the expressions

\[
\begin{align*}
L^{(1)}_{11} &= \psi_1 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - f_{\mu_2} \frac{\partial^2}{\partial x \partial y} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial y} + j \frac{\partial \mu_2}{\partial x} \right) \frac{\partial}{\partial x} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} + j \frac{\partial \mu_1}{\partial y} \right) \frac{\partial}{\partial y} - j \frac{\partial^2 \mu_1}{\partial x^2} + k^2 \frac{\partial \nu_1}{\partial x}, \\
L^{(1)}_{12} &= f_{\mu_2} \frac{\partial^2}{\partial x^2} + (\psi_1 - 1) \frac{\partial^2}{\partial x \partial y} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} + j \frac{\partial \mu_2}{\partial y} \right) \frac{\partial}{\partial x} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial y} + j \frac{\partial \mu_1}{\partial x} \right) \frac{\partial}{\partial y} + \frac{\partial^2 \mu_1}{\partial x^2} + j \frac{\partial^2 \nu_2}{\partial y^2} + j k^2 \frac{\partial \nu_2}{\partial x} + k^2 \frac{\partial \nu_2}{\partial y}, \\
L^{(1)}_{21} &= -f_{\mu_2} \frac{\partial^2}{\partial x^2} + (\psi_1 - 1) \frac{\partial^2}{\partial x \partial y} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} + j \frac{\partial \mu_2}{\partial y} \right) \frac{\partial}{\partial x} + \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} + j \frac{\partial \mu_1}{\partial y} \right) \frac{\partial}{\partial y} - j \frac{\partial^2 \mu_1}{\partial x^2} - j k^2 \frac{\partial \nu_2}{\partial x}, \\
L^{(2)}_{22} &= \frac{\partial^2}{\partial x^2} + \psi_1 \frac{\partial^2}{\partial y^2} + f_{\mu_2} \frac{\partial^2}{\partial x \partial y} - \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} - j \frac{\partial \mu_2}{\partial y} \right) \frac{\partial}{\partial x} - \left( \frac{1}{\varepsilon} \frac{\partial}{\partial x} - j \frac{\partial \mu_1}{\partial y} \right) \frac{\partial}{\partial y} + \frac{\partial^2 \mu_1}{\partial x^2} + \frac{\partial^2 \nu_2}{\partial y^2} + j \frac{\partial^2 \nu_2}{\partial x \partial y} + k^2 \frac{\partial \nu_1}{\partial x}.
\end{align*}
\]

(2)

We shall represent the desired field in the region of an elementary cell \( S_0 \) of the periodic structure in the form

\[ H_0(x, y) = \sum_{i=1}^{2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} V_{i,n,m} f_{i,n,m}, \]

(3)

where \( \{f_{i,n,m}\} \) is a complete system of vector functions which is orthonormal in the region \( S_0 \) satisfying condition (1) and defined by the expressions

\[
\begin{align*}
f_{1,n,+} &= (k_{n,n} x + k_{n,n} y) f_{n,n}, \\
f_{2,n,+} &= (-k_{n,n} x + k_{n,n} y) f_{n,n}, \\
f_{n,n} &= \exp \left[ -j(k_{n,n} x + k_{n,n} y) \right], \\
S_n &= d_1 d_2 \sin \alpha, \\
k_{n,n} &= \sqrt{k_{n,n}^2 + k_{n,n}^2}, \\
k_{n,n} &= (2\pi n + \phi_1)/d_1, \\
k_{n,n} &= (2\pi n + \phi_2)(d_2 \sin \alpha) - (2\pi n + \phi_1)(d_1 \tan \alpha).
\end{align*}
\]

*The wave factor \( \exp[j(\omega t - Bz)] \) is omitted.*