GLOBAL STABILITY OF STATIONARY REGIMES FOR HYDROELECTRIC POWER STATIONS WITH AN EQUALIZING RESERVOIR WHEN THE TURBINE TORQUE IS CONSTANT*

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We consider the problem of global stability of the processes in the pressure system of a hydroelectric power station equipped with a simple equalizing reservoir, under the assumption that the turbine torque is constant and equal to the moment of resistance. The problem is studied using the methods of the qualitative theory of differential equations.

The analysis of global stability of stationary regimes in the pressure arrangements of hydroelectric stations equipped with equalizing reservoirs is usually made under the assumption that so-called "ideal" controls have been established on the turbines, which maintain the mechanical force of the hydraulic machinery at a constant value equal to the opposing forces (a summary of the relevant papers is given in [1], and some additional papers are mentioned in [2]).

In the present paper we consider the problem of global stability of the processes in the pressure system of a hydroelectric station with a simple equalizing reservoir under more general assumptions, namely under the assumption that the turbine torque is constant and equal to the moment of resistance. Under this assumption, the hypothesis of "ideal" controls appears as the special case when there is uniformity of the controls. In this investigation we assume that we have direct current and constant efficiency. The problem is studied using the methods of the qualitative theory of differential equations.

The following results are obtained in this paper: a) it is shown that the boundary of the domain of stability is unsafe; b) two domains of parameter values (among those having practical significance) are found for which the system necessarily has no limit cycles; the system is globally stable for points belonging to one of these, while it is unstable for the other; we determine the qualitative structure of the decomposition of the phase space into trajectories corresponding to these values of the parameters; c) it is shown that the domain of possible initial deviations is smaller for \( \delta = 0 \) (where \( \delta \) is the non-uniformity of the control) than for \( \delta \neq 0 \); the latter result agrees qualitatively with one of the conclusions of [4], obtained in studying stability in the small of the corresponding system; d) it is shown that the domain of global stability of the system becomes substantially larger for \( \delta > \delta_{cr} \).

We consider the pressure system of an isolated hydroelectric installation with a simple cylindrical reservoir feeding an active turbine (we remark that taking into account the flow characteristics of a reactive turbine with the corresponding idealization makes the investigation only slightly more difficult). Suppose that the differential equations describing the processes in the system are the following [3] (see Fig. 1):

The continuity equation:

\[
F \frac{dz}{dt} = fv - Q; \tag{1}
\]

The flow equation, in pressure form (\( v > 0 \)):

\[
L \frac{dv}{dt} + z + P_0 v^2 = 0; \tag{2}
\]

The flow equation for an active turbine:

\[
Q = c_\alpha \sqrt{2g(z + H)}; \tag{3}
\]

The equation for the rotation of the machinery

\[
j \frac{d\omega}{dt} = \eta_1 \frac{Q_0(z + H)}{\omega} - M_c; \tag{4}
\]

The equation for the sensing element (we neglect the mass of the control)

\[
T_k \frac{d\eta}{dt} + \delta_\eta + \varphi = 0; \tag{5}
\]

The servomotor equation (a control with a rigid inverse relation)

\[
T_s \frac{dp}{dt} = f(\varphi - \eta). \tag{6}
\]

Here \( \varphi = (\omega - \omega_0)/\omega_{nom} \) is the relative variation of the angular velocity of the aggregate, \( \varphi = \sigma_\alpha (1 + \eta) \) is the nozzle opening (proportional to the relative displacement of the piston of the servomotor), while the remainder of the notation has the customary meaning. We assume that the servomotor characteristic \( f(z) (z = \eta - \mu) \) is linearizable and satisfies \( f(\varphi) > 0 \) and \( f(0) = 0 \). We also assume that the efficiency of the aggregate is \( \eta_1 = \text{const} \) and the moment of resistance forces is \( M_c = \text{const} \).

Introducing the dimensionless variables \( y = v/\nu_0, x = z/P_0 v_0^2, q = Q/Q_0 \), where the zero subscript stands for quantities in a stationary regime, assuming that \( \omega_{nom} = = \omega_0 \), and setting

\[
\dot{\delta} = \frac{P_0 v_0^2}{H}, \quad T_i = \frac{FP_0 v_0^2}{f v_0}, \quad T_s = \frac{L v_0}{g P_0 v_0^2},
\]

\[
T_3 = \frac{j_0 \nu_0}{\nu_1 Q_0 H}, \quad m_c = \frac{M_c v_0}{\nu_1 Q_0 H} = \text{const}, \quad T_4 = T_k, \quad T_s = T_1,
\]

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we rewrite Eqs. (1)-(6) in the following form:

\[
\begin{align*}
T_1 \frac{dx}{dt} &= x - q , \quad (1a) \\
T_2 \frac{dy}{dt} + x + y^2 &= 0 , \quad (2a) \\
q &= c \frac{\alpha}{Q_0} \sqrt{2gH(1 + \beta x)} , \quad (3a) \\
T_3 \frac{d\varphi}{dt} &= \frac{q(1 + \beta x)}{\omega_1} - m_\varphi , \quad (4a) \\
T_4 \frac{d\eta}{dt} + \delta \eta + \varphi &= 0 , \quad (5a) \\
T_5 \frac{d\mu}{dt} &= f(\eta - \mu) , \quad (6a)
\end{align*}
\]

where \( \mu = (x - a_0)/\omega_1, 1 + \beta x > 0 \) (since the level of the fluid in an equalizing reservoir cannot be lower than its bottom). We assume that the time constants \( T_i \) satisfy the following inequality:

\[
T_i , T_2 \ll T_3 \ll T_4 , T_5 . \quad (7)
\]

In connection with Eq. (7) we can regard part of the differential equations of the problem as containing a small parameter in front of the derivative, and the motions in the system can be separated into the fastest (\( \eta, \mu \)), the less fast (\( \varphi \)), and the slow motions \( (x, y) \) [6]. The approximate equations for the fastest motions are

\[
\begin{align*}
T_1 \frac{d\eta}{dt} + \delta \eta + \varphi &= 0 \quad (\varphi = \text{const}) , \quad (8) \\
T_4 \frac{d\mu}{dt} &= f(\eta - \mu) . \quad (9)
\end{align*}
\]

The points in the phase spaces with \( \delta \eta + \varphi = 0 \) and \( \eta - \mu = 0 \) are (for the given servomotor characteristic, and for \( \delta > 0 \)) stable equilibrium conditions of the fastest motions. Hence if at the initial time the representative point is not in the vicinity of these subspaces then, moving along the trajectories of the fastest motions, it reaches the subspaces and will subsequently move inside them. In light of Eqs. (3a), (4a) the approximate equation for the less fast motions is of the form

\[
\begin{align*}
T_3 \frac{d\varphi}{dt} &= \frac{c_\varphi(1 - \varphi) - 1}{\omega_1} \sqrt{2gH(1 + \beta x)} (1 + \beta x) - m_\varphi = \\
&= F_\varphi(x, \varphi) \quad (x = \text{const}) , \quad (10)
\end{align*}
\]

where \( \mu = \eta = -\varphi/\delta_1, \omega_1 = 1 + \varphi \). The equilibrium state for these motions will be the subspace \( F_\varphi(x, \varphi) = 0 \),

where we can easily verify that \( \frac{\partial F_\varphi}{\partial \varphi} < 0 \), and hence this equilibrium state is stable. From the equation

\[
F_\varphi(x, \varphi) = 0 . \quad (11)
\]

we obtain

\[
\varphi = \frac{c_\varphi \sqrt{2gH(1 + \beta x)} (1 + \beta x) - m_\varphi Q_0}{m_\varphi Q_0 + c_\varphi \delta_1 - \frac{1}{2} \sqrt{2gH(1 + \beta x)} (1 + \beta x)} . \quad (12)
\]