Waves in Partially Saturated Porous Media

D. M. J. SMEULDERS, J. P. M. DE LA ROSETTE, and M. E. H. VAN DONGEN
Department of Physics, Eindhoven University of Technology, PO Box 513, NL-5600 MB Eindhoven, The Netherlands

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Abstract. The propagation of compressional waves in a porous medium is investigated in case the pore liquid contains a small volume fraction of gas. The effect of oscillating gas bubbles is taken into account by introducing a frequency-dependent fluid bulk modulus, which is incorporated in the Biot theory. Using a shock tube technique, new experimental data are obtained for a porous column subjected to a pressure step wave. An oscillatory behaviour is observed, consisting of two distinct frequency bands, which is predicted by the theoretical analysis.

Key words. Waves, dispersion, shock tube, gas bubbles.

1. Introduction

Although a theoretical description of wave propagation in saturated porous media was already available in 1956 (Biot), it was not until 1980 that first experimental verification of the most crucial aspect of the theory was reported when Plona actually observed the second compressional wave (Plona, 1980). Since then, the Biot theory has been successfully applied to various problems such as fourth sound in a superfluid system (Johnson, 1980) and fluid diffusion through elastic porous media (Chandler, 1981). Soon, acoustical properties of fully liquid saturated and gassy porous media were observed to be significantly different. An extended literature survey has been given by Anderson and Hampton (1980).

In recent years, the two-phase Biot theory has been modified to allow for the presence of a third phase, viz. the gas phase. Garg and Nayfeh (1986) and Berryman et al. (1988) present rather general models, applicable to a wide variety of gas volume fractions. Bedford and Stern (1983) describe the effect of a small amount of gas bubbles on the propagation and damping of the two compressional waves. Experimental data are given by Dontsov et al. (1987), Van der Grinten et al. (1988), and Sniekers et al. (1989) who all use a shock tube technique.

2. Theory

In 1956, Biot proposed a straightforward phenomenological theory of wave propagation in saturated porous media. It is assumed that volumes can be identified, large when compared to the pore and grain size but small when compared to the
wavelength, and that each volume element is described by the average displacement of the fluid $U(r,t)$ and the solid parts $u(r,t)$.

The equations of motion are

$$GV^2u + \text{grad}[(A + G)Vu + QVu] = \frac{\partial^2}{\partial t^2} (\rho_{11}u + \rho_{12}U) + b \frac{\partial}{\partial t} (u - U),$$  \hspace{0.5cm} (1a) 

$$\text{grad}[QVu + RVU] = \frac{\partial^2}{\partial t^2} (\rho_{11}u + \rho_{12}U) - b \frac{\partial}{\partial t} (u - U).$$  \hspace{0.5cm} (1b)

For cases in which the skeletal frame is a homogeneous medium, and in the absence of locked-in voids (all pores are interconnected), $A$, $Q$, and $R$ are generalized elastic coefficients which can be related to the bulk modulus of the fluid $K_f$, the bulk modulus of solid $K_s$, the bulk modulus of skeletal frame $K_b$, and to $G$ which is the shear modulus of the skeletal frame

$$A = \frac{(1 - \varphi)^2 K_s - (1 - 2\varphi)K_b + \varphi K_b \left(\frac{K_s}{K_f} - 1\right)}{1 - \varphi - \frac{K_b}{K_s} + \varphi \frac{K_s}{K_f}} - \frac{2}{3} G,$$  \hspace{0.5cm} (2a) 

$$Q = \frac{\left(1 - \varphi - \frac{K_b}{K_s}\right)\varphi K_s}{1 - \varphi - \frac{K_b}{K_s} + \varphi \frac{K_s}{K_f}},$$  \hspace{0.5cm} (2b) 

$$R = \frac{\varphi^2 K_s}{1 - \varphi - \frac{K_b}{K_s} + \varphi \frac{K_s}{K_f}},$$  \hspace{0.5cm} (2c)

where $\varphi$ is the porosity (fluid volume fraction).

Equations (2) are equivalent to those given by Stoll (1974) or Geertsma and Smit (1961). The density terms $\rho_{ij}$ are related to the density of solid $\rho_s$ and fluid $\rho_f$ by

$$\rho_{11} = (1 - \varphi)\rho_s - \rho_{12},$$  \hspace{0.5cm} (3a) 

$$\rho_{22} = \varphi \rho_f - \rho_{12}.$$  \hspace{0.5cm} (3b)

The coefficient $\rho_{12}$ represents a mass coupling parameter between fluid and solid. It is always negative and proportional to the fluid density:

$$\rho_{12} = - (\alpha - 1)\varphi \rho_f,$$  \hspace{0.5cm} (4)

where $\alpha > 1$ is often referred to as tortuosity parameter; a purely geometrical quantity independent of solid or fluid density.

The remaining parameter $b$ describes the frequency-dependent interaction force between fluid and solid. At lower frequencies, $b$ will show a Stokes flow behaviour,