The remaining integral does not contribute to the pole term.

Collecting together all the terms, we have

$$J_1 = \frac{1}{\varepsilon}\int_0^1 \frac{dx\, x(5x-3)}{\sqrt{1-x}} + \frac{4}{\varepsilon} + O(1) = \frac{16}{3}\frac{1}{\varepsilon} + O(1).$$

Calculating similarly the asymptotic behavior of J_2, we finally obtain $I_1 = 8\pi^3 m/\varepsilon$. For I_2, integrating by means of Eqs. (A.1) and (A.2), we obtain $I_2 = 2\pi^3/m\varepsilon$. Thus,

$$\mathrm{P.P.}\,\tilde{\Gamma}_1^{(1)} = -\frac{m}{\pi^3\varepsilon} + \frac{A_2 m}{8\pi^2\varepsilon}.$$

Besides the graph of Fig. 4c, it is necessary to take into account the contribution to $\Gamma_1^{(1)}$ of the graphs of Figs. 4d and 4e, in which the cross denotes the vertices corresponding to P.P. $\Gamma_3^{(1)}$ and P.P. $\Gamma_2^{(1)}$. The calculation shows that their contribution to $\Gamma_1^{(1)}$ completely compensates the contribution of the graph of Fig. 4c and, therefore, $a_{01} = 0$.

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GENERALLY COVARIANT THEORIES OF GAUGE FIELDS ON SUPERSPACE

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Different variants of a generally covariant theory of superfields with nonzero values of the curvature and torsion tensors are discussed from the point of view of the holonomy group. A study is made of the example of a Lagrangian that is quadratic in the torsion tensor in the linear approximation of weak fields with interaction switched off and includes free fields with spin 2 and Rarita–Schwinger fields with spin 3/2.

1. After the introduction of the concept of supersymmetry [1-3] and the construction of the simplest supersymmetric theories as field theories on superspace [4-7] the problem arose of finding gauge and generally covariant generalizations of these theories. The searches for such generalizations have been made and are currently being made in essentially two directions, depending on the type of projection operators that

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are used to separate the physical and unphysical states.

In one of these directions, one uses projection operators that directly select a value of the superspin of particles in such a way that superfunctions with other values of the superspin play the role of gauge transformations. In this direction, we have the important papers of Ferrara and Zumino [8] and Salam and Strathdee [9], in which a supersymmetric generalization of the Yang–Mills theory was obtained, and of Ogievetskii and Sokatchev [10], who use a vector superfield to construct a supersymmetric generalization of the theory of gravitation. The recent papers of Freedman, van Niewenhuizen, and Ferrara and Deser and Zumino [11], in which a unified supersymmetric description of the gravitational field and the Rarita–Schwinger field is proposed, belong, as regards their physical content, to this direction, although from the technical side the construction of gauge transformations with projection operators corresponding to a definite value of the superspin was not undertaken.

The other direction in the search for gauge and generally covariant generalizations of supersymmetric theories proceeds from attempts to construct various kinds of connection on superspaces in close analogy with the schemes that use connections to describe the Yang–Mills and Einstein fields in ordinary space. The projection operators that then arise are related to the operation of exterior differentiation. Such an approach was proposed for the first time in [12] with the only exceptional feature that the action integral was taken in the form of an invariant integral over a four-dimensional surface in superspace. It is obvious that the possibility is not precluded of taking the action integral in the form of an invariant integral over a six-dimensional surface in superspace.

The direct generalization of Einstein's equations to the case of an eight-dimensional Riemannian superspace was proposed by Arnowitt, Nath, and Zumino [13, 14] and somewhat later for the case of an arbitrary connection with nonzero curvature and torsion tensors by Zumino [15] and the present authors [16].

The basic ideas relating to the possibility of constructing the theory of a gauge superfield in closer analogy with the ordinary theory of Yang–Mills fields were recently formulated in the review [17] by Ogievetskii and Mesincescu, and a concrete variant of such a theory was proposed in [18] by Ogievetskii and Sokatchev. As is noted in [18], the proposed variant of the theory of the gauge superfield is clearly not free of difficulties associated with the appearance of redundant field components when an interaction is present.

As will be shown in the present paper, similar difficulties arise when one considers generally covariant theories, and the question of their consistent elimination is at the present open.

In this paper, in more detail than in [15, 16], we consider the formalism for introducing connections on superspace, emphasizing the importance for physical applications of the specification of a definite holonomy group. * We consider examples of generally covariant action integrals and some of their consequences.

2. An arbitrary superspace with coordinates $z^a=(x^\mu, \varphi^\alpha, \varphi^{\dot\alpha})$ is described by the Cartan structure equations [16]

$$d\wedge\omega^A(\delta)+\omega^B(\delta)\wedge\Gamma_B{}^A(d)={}^1\!/\!_2\,\omega^B(\delta)\wedge\omega^C(d)S_{CB}{}^A, \tag{1}$$

$$d\wedge\Gamma_A{}^B(\delta)+\Gamma_A{}^C(\delta)\wedge\Gamma_C{}^B(d)={}^1\!/\!_2\,\omega^C(\delta)\wedge\omega^D(d)R_{DC,\,A}{}^B, \tag{2}$$

where the differential forms $\omega^A(d)=dz^a\omega_a{}^A$ define the transition from the natural local coordinate system associated with the coordinates $z^a=(x^\mu, \varphi^\alpha, \varphi^{\dot\alpha})$ to an arbitrary local frame (we shall denote the components of tensors in an arbitrary local frame and in a natural local coordinate system by upper case and lower case letters, respectively), $\Gamma_A{}^B(d)$ are the differential forms of the connection, and $S_{CB}{}^A$ and $R_{DC,\,A}{}^B$ are the torsion and curvature tensors. † The differentiations and products of forms in the expressions (1) and (2) are exterior and are defined in the same way as for the case of ordinary spaces by alternation of the differentials d and δ.

The form of the torsion and curvature tensors follows from Eqs. (1) and (2):

$$S_{CB}{}^A=[(-)^{C(B+f)}X_B{}^fX_C{}^l\partial_l\omega_f{}^A+\Gamma_{CB}{}^A]-(-)^{CB}[B\leftrightarrow C], \tag{3}$$

$$R_{DC,\,B}{}^A=[(-)^{D(C+f)}X_C{}^fX_D{}^l\partial_l\Gamma_{fB}{}^A+(-)^{D(C+B+F)}\Gamma_{CB}{}^F\Gamma_{DF}{}^A]-(-)^{CD}[C\leftrightarrow D], \tag{4}$$

* The holonomy group is the group of transformations of the frame when it is taken in parallel transport around an infinitesimally small closed contour.

† The invariant contraction with respect to tensor indices is defined in the Appendix.