A STEADY CONICAL INSTABILITY REGIME OF PLASMA WAVES

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The steady conical instability regime of longitudinal plasma waves ($\omega = \omega_{pe}$) with a constantly acting source of energetic electrons is discussed in the quasilinear approximation. It is shown that with the relativistic effects which appear at sufficiently small values of the longitudinal wavenumber $k_l$ taken into account the spectrum of the excited waves is discrete, i.e., waves with fixed $|k_l|$ and $k$ are excited. An estimate of the concentration of energetic electrons in the magnetic trap as well as the relationships which connect the energy density of the plasma waves to the power of the source of energetic particles are obtained. The numerical estimates correspond to the solar corona parameters.

For many astrophysical applications the problem of a steady conical instability regime in a magnetic trap with a constantly acting source of energetic electrons is of interest. This problem has been discussed with application to the plasma of the earth's magnetosphere in a number of papers (see [1]) for instability by whistlers and in [2, 3] for the instability by plasma waves. In the case of instability by whistlers the steady regime is provided by a constant flux of particles into the loss cone due to their diffusion by pitch angles upon interaction with waves. In the case of instability by plasma waves the steady regime is provided by the drift of particles across the trap, which compensates the heating of the electrons in the transverse direction upon the interaction of the latter with plasma waves. However, as is shown below, as a result of the development of a conical instability plasma waves are excited even with small but finite values of the longitudinal component of the wave vector, and therefore diffusion by pitch angles will occur along with diffusion by energies. This means that a steady conical instability regime of plasma waves can occur without taking...
into account the drift of the particles across the trap. This situation is most probable for magnetic traps on the sun (arched magnetic configurations), in which the characteristic dimensions of inhomogeneity of the magnetic field are very large. This instability regime can, in our opinion, also occur in the magnetosphere of the earth as well as in laboratory experiments.

Let a trap be filled with cold (the main) plasma and let a constantly acting source inject energetic electrons into it. We shall assume, for simplicity's sake, the magnetic field to be uniform over the entire length of the trap with the exception of the regions near the plugs. The quasilinear system of equations describing the interaction of particles with the plasma waves has the form [4, 5]

\[
\frac{\partial}{\partial t} \left( D_\parallel \frac{\partial f}{\partial v_\parallel} + D_\perp \frac{\partial f}{\partial v_\perp} \right) = -I,
\]

\[
D_\parallel \frac{\partial}{\partial t} \left( \frac{1}{v_\parallel} \frac{\partial f}{\partial v_\parallel} \right) + D_\perp \frac{\partial}{\partial t} \left( D_\parallel \frac{\partial f}{\partial v_\parallel} + D_\perp \frac{\partial f}{\partial v_\perp} \right) = -I,
\]

in a cylindrical coordinate system in the approximation of a weak magnetic field (ωHe << ωpe, ωHe << k_Lν_L, ωHe << k_Tν_T). Here f is the distribution function of the energetic electrons; ωHe, electron cyclotron frequency; ωpe, electron plasma frequency; v_L and v_T, longitudinal and transverse components of the velocity of the electrons with respect to the external magnetic field; k_L and k_T, longitudinal and transverse components of the wave vector; e, electron charge; m, electron mass; Wk, spectral energy density of the plasma waves; n, electron concentration of the main plasma; I, source function of the energetic electrons; γ, growth increment of the waves; and ν, attenuation decrement of the waves.

A steady regime occurs when the quasilinear growth increment of the waves γ and their attenuation decrement are equal, for example, due to Coulomb collisions, ν. The fundamental difference from a conical instability by whistlers consists of the fact that here a non-one-dimensional spectrum of plasma waves is excited and waves with a wavenumber fixed in modulus interact with practically all the electrons. Since, as is shown below, the quasilinear increment has a maximum at some k_L ≠ 0 and k_T = ωpe/v_Te (v_Te is the thermal velocity of the electrons of the main plasma), it is natural that precisely this maximum value enters into the equality γ = ν. All the remaining modes of the plasma waves have a smaller increment and die out. Thus the spectrum of plasma waves in a steady conical instability regime should be discrete, i.e., it should have fixed values of the wavenumbers k_L and k_T. In addition the time-dependent distribution function of particles in the trap should be an even function of v_T, and the steady-state spectrum of the waves should have two discrete values of k_T which differ in sign. One should note that we will discuss the regime of weak diffusion [1], in which τ_D > στ_0, where τ_D is the characteristic diffusion time, σ is the corkscrew ratio (the ratio of the maximum value of the magnetic field in the trap to the minimum value), τ_0 = L/v_L, L is the length of the trap, and v_L is the characteristic longitudinal velocity of the particles. In this case in the course of diffusion the loss cone does not have time to fill up, and one can assume it to be empty. With a specified trap length this imposes definite restrictions on the power of the source of energetic electrons. With more powerful sources a regime of moderate or strong diffusion is realized in which the spectrum of the excited waves is evidently more broadband.

And so we shall consider a steady conical instability regime in which plasma waves with fixed values of the wavenumbers |k_L| and k_T are excited. The characteristics of the equation \([(ωpe - k_Tv_T)/v_T]|\partial f/\partial v_T + k_T|\partial f/\partial v_L = 0\), which are circles, are illustrated in Fig. 1. They are the diffusion lines of particles in the fields of each of the waves with fixed k_L and k_T [5]. The boundary of the loss cone is denoted by the dashed line. We shall consider the case