ELECTROMAGNETIC-WAVE REFLECTION FROM A NONLINEAR DIELECTRIC PLATE IN A WAVEGUIDE

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The interaction of opposing TE-waves in a nonlinear cubical dielectric located in a waveguide of arbitrary cross-section is examined. It is shown that the propagation constants of the interacting waves are functions of the combination of the amplitudes of those waves. Wave reflection and refraction in a nonlinear plate are investigated. A formula for the reflection coefficient and a "transillumination" condition of the nonlinear plate are obtained.

The wave reflection and refraction of a nonlinear plate are of great interest in many experimental problems, since under real conditions we are working with media of finite dimensions. Harmonic generation and frequency shift occur in plates of a nonlinear dielectric. For an unbounded linear plate, reflection and refraction have been studied fairly well [1-3]. Similar problems for a plate of a nonlinear dielectric of finite dimensions in a waveguide remain practically unsolved. The reflection of a TE-wave from a plate of a nonlinear dielectric with cylindrical boundaries in a rectangular waveguide has been examined [4], but the dependence of the propagation constant on wave amplitude was not taken into account.

As was shown earlier [5], for electromagnetic-wave propagation in a waveguide, the propagation constant is a function of the electric-field strength of the wave. Naturally, for two or more waves, their propagation constants and, therefore, their phase velocities will be functions of the combination of the electric fields of the waves. We shall examine this effect using the example of two opposing waves in a waveguide filled by a nonlinear medium with dielectric constant

\[ \varepsilon(\omega) = \varepsilon_0(\omega) + 2\alpha(\omega) \mathbf{E}^2. \]  

This case is interesting in that its study makes it possible to solve the problem of wave reflection from a nonlinear dielectric plate in a waveguide. The entire examination will be performed assuming that the nonlinearity of the dielectric is small, using the procedure and designations described earlier [5].

We specify the field of a TE-wave by the potential \( H_z \):

\[ H_z = H_{zn}^{(0)} + \alpha H_{zn}^{(1)}, \]

where \( H_{zn}^{(0)} \) and \( H_{zn}^{(1)} \) zeroth and first approximations, and represent \( H_{zn}^{(0)} \) as

\[ H_{zn}^{(0)} = \left\{ \hat{B}^{(+)}_n e^{i\gamma_1^+ z} + \hat{B}^{(-)}_n e^{-i\gamma_1^- z} \right\} \psi_n(x, y), \]

where \( \hat{B}^{(\pm)}_n \) are the amplitudes of the forward and backward waves and \( \gamma_1^{(\pm)} \) are their propagation constants. We note that, generally speaking, \( \gamma_1^{(+)} \neq \gamma_1^{(-)} \), since each is a function of the electric fields and, in this sense, a nonlinear waveguide does not possess the property of reciprocity.

We shall seek the dependence of the propagation constants on the electric fields of the waves in the form

\[ \gamma_n^{(\pm)} = k_n^2 - \lambda_n^2 + a \gamma_n^{(\pm)}. \]
where $\hat{g}_n(\pm)$ are the unknown and desired functions of the electric-field amplitudes, $k_0 = \omega/c$, and $\hat{\psi}_n$ and $\hat{\lambda}_n$ are the eigenfunctions and eigenvalues of the second boundary-value problem for the waveguide cross-section.

Potential $H_z$ satisfies the equation

$$\Delta H_z + k_0^2 c H_z - k_0^2 x^2 \left( \nabla^2 H_z \right) = 0. \tag{4}$$

Amplitudes $\hat{B}_n(\pm)$ are, generally speaking, complex and we represent each as

$$\hat{B}_n(\pm) = \hat{B}_n(\pm) e^{i\hat{\beta}_n(\pm)} \tag{5},$$

where $\hat{\beta}_n(\pm)$ are the corresponding phases.

The dielectric constant of the medium (1) in these terms has the form

$$\varepsilon = \varepsilon_0 + \alpha \left( |\hat{B}_n(\pm)|^2 + |\hat{B}_n(\mp)|^2 \right. +$$

$$+ 2 |\hat{B}_n(\pm)| |\hat{B}_n(\mp)| \cos \left[ \left( \gamma_n(+) + \gamma_n(-) \right) z + \beta_n \right] \right) \varepsilon_0 \left( \hat{\psi}_n(\pm) \right)^2,$$

where $\hat{\beta}_n = \hat{\beta}_n(\pm) - \hat{\beta}_n(\mp)$ and $\varepsilon$ is written using only a zeroth approximation (2). Then we must calculate the value that determines the right side of the wave equation for $H_z(\pm)$

$$\left( \Delta + k_0^2 c \right) H_z(\pm) = \left[ \gamma_n(\pm) \hat{B}_n(\pm) e^{i\gamma_n z} + \gamma_n(\mp) \hat{B}_n(\mp) e^{i\gamma_n z} \right] \hat{\psi}_n(x, y) +$$

$$+ \left[ \hat{B}_n(\pm) e^{i\gamma_n z} + \hat{B}_n(\mp) e^{-i\gamma_n z} \right] \times$$

$$\times \left( |\hat{B}_n(\pm)|^2 + |\hat{B}_n(\mp)|^2 + 2 |\hat{B}_n(\pm)| |\hat{B}_n(\mp)| \cos \left[ \left( \gamma_n(+) + \gamma_n(-) \right) z + \beta_n \right] \right) \hat{\phi}_n(x, y),$$

where $\hat{\phi}_n(x, y)$ has the form

$$\hat{\phi}_n(x, y) = k_0^2 \lambda_n \nabla^2 \left( \hat{\psi}_n(\pm) \hat{\psi}_n(-) \right)^2.$$

Solution of the problem of finding a first approximation now comes down to the solution of three inhomogeneous linear wave equations for the terms of the sum

$$H_z^{(1)} = H_z^{(+)} + H_z^{(-)} + \bar{H}_z$$

of the form

$$\Delta H_z^{(\pm)} + \left( \gamma_n(\pm)^2 + \gamma_n(-)^2 \right) H_z^{(\pm)} =$$

$$= \hat{B}_n(\pm) \left[ g_n(\pm) - k_0^2 \lambda_n \left( |\hat{B}_n(\pm)|^2 + 2 |\hat{B}_n(\mp)|^2 \right) \right] \hat{\psi}_n e^{i\gamma_n z} -$$

$$- k_0^2 \lambda_n \hat{B}_n(\pm) \left( |\hat{B}_n(\pm)|^2 + 2 |\hat{B}_n(\mp)|^2 \right) \sum_{m=1}^{\infty} \mu_{nm}(x, y) e^{2i\gamma_n z}.$$