A small strain, moderately large deflection finite element beam model with cross-sectional warping effect

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Abstract. A finite element model that can be applied to helicopter rotor blades has been developed with a particular emphasis on the proper representation of out-of-plane warping of arbitrary cross-sections. The model can describe accurately coupled bending, torsion and extensional behavior of beams undergoing small strain, moderately large deflection. The model can also handle beams with arbitrary cross-sections, taper, pretwist and planform. A main feature of the present approach is to introduce small warping displacement superimposed over flat cross-sections of a shear-flexible beam in the direction of the reference axis in deformed configuration. The nonlinear equilibrium equation resulting from finite element approximation is solved by the Newton-Raphson method. Numerical tests involving simple isotropic beams demonstrate the validity of the present approach.

1 Introduction

Beams with complicated cross-sections, taper, pretwist and curved planform may exhibit coupling among extensional, bending and torsional behavior (Dowell, Traybar and Hodges 1977; Rosen 1980; Hodges 1980; Krenk 1983; Peterson 1982; Rosen 1983; Hodges, Ormiston and Peters 1986). Helicopter rotor blade is a typical example of beams with complicated geometries. Recently helicopter blades are made out of composite materials for improved aeroelastic performance. Composite with different ply layups exhibits stiffness coupling among extension, bending and torsion. In order to properly represent these coupling behaviors, the effect of warping should be taken into account. Moreover, helicopter blades are flexible and experience moderately large deflection under operational condition.

Many researchers have developed finite element models including warping effects in various ways for small or moderately large deflection (Wekezer 1984; Krenk and Gunneskov 1981; Christensen and Lee 1986; Hong and Chopra 1986; Bauchau and Hong 1987). However, these types of finite element models can not represent accurately the cross-sectional warping of beams with complicated geometries. Recently finite element beam models that can handle arbitrary large rotations have been developed (Iura and Atluri 1988; Simo and Vu-Quoc 1986). However, these models do not deal with warping of arbitrary cross-sections.

Of course, beams can be modelled with shell elements. For example, a nine node shell element based on the degenerate solid shell concept and the mixed formulation (Rhiu and Lee 1987) can be used for an accurate modeling of beams. However, in comparison with a beam model, a shell element model requires more degrees of freedom and consequently more computing time. In addition, for helicopter rotor aeroelastic problems in which aerodynamics is coupled with structural stiffness, it is more convenient to model rotor blades as beams rather than as shells.

An advanced finite element formulation was developed to model beams with warping effects by introducing small warping displacements superimposed over a flat cross-section of a shear-flexible beam (Kim and Lee 1987). The formulation can model coupling among extensional, bending and torsional behavior of beams with complicated cross-sections, taper, pretwist and curved planform. Numerical tests demonstrated the validity of the approach. However, the scope of the work was limited to small displacement and isotropic elastic material. In the present paper the approach (Kim
and Lee 1987) is extended to incorporate moderately large deflection. However, strain is still assumed to be small. Although the ultimate goal of the ongoing study is to develop a formulation which can be used to model composite rotor blades, the present study is limited to the isotropic elastic case.

2 Formulation

The geometry and the kinematics of deformation will be described by the Lagrangian formulation with the original undeformed configuration as the reference configuration. Figure 1 shows a portion of a beam in the original undeformed configuration and in the deformed configuration. In the figure, the line connecting point O on the reference axis and a generic material point P is embedded in the cross-section which is normal to the reference axis. Note that the location of the reference axis is arbitrary except that it is normal to the cross-section. For convenience, a local orthogonal coordinate system is defined at point O in addition to the global Cartesian coordinate system. The global coordinate system has components $X$, $Y$ and $Z$ along the direction of unit vectors $i_1$, $i_2$ and $i_3$. The local coordinate system with components $x$, $y$ and $z$ is defined such that unit vector $a_1$ along $x$ direction or the reference axis is normal to the undeformed cross-section, while unit vectors $a_2$ and $a_3$ are parallel to $y$ and $z$ axis whose directions can be chosen arbitrarily. Then the position vector of point $P$ before deformation can be expressed as

$$R = R_0 + ya_2 + za_3$$

(1)

where $R$ is the position vector of point $P$ with components $X$, $Y$ and $Z$ in the global coordinate system and $R_0$ is the position vector of point $O$ with components $X_0$, $Y_0$ and $Z_0$ in the global coordinate system.

A flat cross-section which is initially normal to the $a_1$ axis is assumed to be rigid in its own plane. This assumption is acceptable for many beam-like structures such as helicopter rotor blades. The section translates and rotates in the three dimensional space and the unit vectors $a_1$, $a_2$ and $a_3$ rotate to $a'_1$, $a'_2$ and $a'_3$. In addition, small warping displacement vector $fA_1$ is superposed over the flat cross-section in the direction of $A_1$ which is a vector parallel to the references axis in deformed configuration. The $A_1$ vector need not to be a unit vector as will be shown later in Eq. (6). Note that the direction of $A_1$ is slightly different from that of the unit vector $a_1$ because of the transverse shear strain as shown in Fig. 2. Then the displacement vector $U$ of point $P$ with three components $U$, $V$ and $W$ in the global coordinate system is expressed as

$$U = u + y(a'_2 - a_2) + z(a'_3 - a_3) + fA_1$$

(2)