FORMAL AND MATERIAL CONSEQUENCE

1. INVALIDITY

How do we show that an argument is invalid? Consider this example:

(1) All cats are animals
    Some animals have tails
    So some cats have tails.

The premises are true and so is the conclusion. Yet there is an obvious sense in which the truth of the premises does not guarantee that of the conclusion. The argument is invalid. But how can we show that invalidity?

One thought is, that arguments of the same sort, or form, actually lead from truth to falsity. Although their premises are true, their conclusions are false. The same could have been true of (1), though in fact it isn't. What we might try, therefore, is to formalise the argument, and show that the form is invalid. Using the above example, we obtain

(2) \((\forall x)(Fx \rightarrow Gx), (\exists x)(Gx \land Hx) \vdash (\exists x)(Fx \land Hx)\),

where \(Fx\) reads ‘\(x\) is a cat’ , \(Gx\) is ‘\(x\) is an animal’ and \(Hx\) is ‘\(x\) has a tail’. To show this form is invalid, we find another instance of it, with a different key, but in which, though the premises are still true, the conclusion is false. For example, we might let \(Fx\) and \(Gx\) read as before, but let \(Hx\) read ‘\(x\) is a dog’:

(3) All cats are animals
    Some animals are dogs
    So some cats are dogs.

The premises are true and the conclusion false. So this argument really is invalid. Since every instance of a valid form is valid, (2) is an invalid form.
Does this show that (1) is invalid? Not immediately; for every valid argument is in instance of some invalid form. For example, every two premise argument is an instance of the form

(4) \( P, Q, \vdash R \),

which is patently invalid. But that does not show that every two-premise argument is invalid.

The trouble with (4), of course, is that it does not reveal sufficient structure. What we try to do when we formalise an argument like (1) is to articulate its structure so that if there is a dependency of the conclusion on the premises it will be revealed. Such a technique is ideal when we find a form which is valid and of which the argument is an instance. But what if we cannot – as with (1)?

What we may be tempted to say is that (2) reveals as much structure in (1) as can be revealed. Since (2) is invalid, this means that (1) has failed its best possible chance to be shown valid, and so must be invalid.

It does show that (1) is not valid in virtue of its form. But does that show it is not valid? How else might it be valid? In a valid argument, the truth of the premises must somehow rule out the falsity of the conclusion. So it must be impossible for the premises to be true and the conclusion false. Could the premises of (1) be true and the conclusion false?

Suppose the world were much as it is now, but cats evolved to become tailless – Manx cats take over, say. In such a world, all cats are animals, some animals have tails (cats no longer do, but dogs are unchanged), but now no cats have tails. We have represented to ourselves a situation in which the premises are true and the conclusion false. So the truth of the premises does not guarantee that of the conclusion. (1) is invalid.

John Etchemendy (1990) contrasts "interpretational” with “representational semantics”. In representational semantics we describe a situation, perhaps different from how things actually are, in which the propositions take various values. In interpretational semantics, we interpret certain expressions differently from their actual interpretation to much the same effect. When we formalised (1) as (2) and then interpreted the predicate letters in (2) to obtain (3), we varied the interpretation – we effectively interpreted ‘have tails’