HYPERSONIC FLOW PAST BLUNT-EDGED DELTA WINGS

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A method is proposed for calculating hypersonic ideal-gas flow past blunt-edged delta wings with aspect ratios $\lambda = 100-200$. Systematic wing flow calculations are carried out on the intervals $6 \leq M_\infty \leq 20$, $0 \leq \alpha \leq 20^\circ$, $60^\circ \leq \chi \leq 80^\circ$; the results are analyzed in terms of hypersonic similarity parameters.

It is proposed to consider the steady-state hypersonic flow of an ideal gas past a delta wing with a spherical nose and cylindrical leading edges.

A number of theoretical and experimental studies have been devoted to this problem [1–3]. Among them we note Lunev's monograph [2], in which the gas flow divergence effect in the neighborhood of the plane of symmetry of the wing was first established analytically.

In [4, 5] numerical methods and algorithms for solving this problem were developed; however, the numerical results were obtained for small wing aspect ratios $\lambda = x/r_0 \leq 10$ (Fig. 1) on a limited interval of the governing parameters of the problem — the Mach number $M_\infty$, the sweep angle $\chi$, and the angle of attack $\alpha$.

We will examine the steady-state supersonic flow of an ideal gas past a delta wing with cylindrical edges and a spherically blunted nose (Fig. 1). In the flow field we distinguish three characteristic zones: the nose bluntness zone (I); the near-edge zone with high gradients of the gas dynamic parameters (II); and the zone over the flat windward and leeward sides of the wing (III).

In each of zones I–III we introduce a specific coordinate system: spherical, cylindrical, and Cartesian, respectively.

The equations describing the steady-state supersonic ideal-gas flow in these zones have the form:

$$\frac{d}{dq_3} \int_S \left[ \begin{array}{c} \rho \\ \rho u \\ \rho v \\ p \\ w \\ w \\ w \\ w \\ w \end{array} \right] = \oint_{\Gamma} \left[ \begin{array}{c} \frac{\partial}{\partial \xi} \left( \rho u \delta \xi \right) \\ \frac{\partial}{\partial \eta} \left( \rho v \delta \eta \right) \\ \frac{\partial}{\partial \zeta} \left( \rho w \delta \zeta \right) \\ \frac{\partial}{\partial \xi} \left( \rho \right) \\ \frac{\partial}{\partial \eta} \left( p \right) \\ \frac{\partial}{\partial \zeta} \left( p \right) \\ \frac{\partial}{\partial \xi} \left( w \right) \\ \frac{\partial}{\partial \eta} \left( w \right) \\ \frac{\partial}{\partial \zeta} \left( w \right) \end{array} \right]$$

Here, $\Gamma$ is a closed contour bounding the area element $S$ on the surface $q_1 = \text{const}$, the vector $\xi = \partial n / \partial q_1$, where $\partial n$ is the projection of the displacement of $\Gamma$ onto its outward normal $n$, and $\delta \xi$ and $\delta \eta$ are the components of the vector $\xi$ along the axes $q_2$ and $q_3$; $q_1$, $q_2$, $q_3$ is a generalized orthogonal coordinate system.

In the Cartesian coordinate system

$$q_1 = x, \quad q_2 = y, \quad q_3 = z, \quad \alpha_0 = b_0 = c_0 = 1, \quad f = 0$$

In the cylindrical system

$$q_1 = x, \quad q_2 = r, \quad q_3 = \varphi, \quad a_0 = b_0 = 1, \quad c_0 = r, \quad f = \frac{\rho}{r} \left[ \begin{array}{c} \nu \\ w \\ w \\ w \end{array} \right]$$

In the spherical system

$$q_1 = x, \quad q_2 = r, \quad q_3 = \theta, \quad a_0 = b_0 = c_0 = 1, \quad f = \frac{\rho}{r} \left[ \begin{array}{c} \nu \\ w \end{array} \right]$$

The system of equations (1) is closed by the equation of constancy of the total enthalpy

\[ \frac{2\gamma}{\gamma-1} \frac{\rho}{\rho} + \frac{u^2+v^2+w^2}{(\gamma-1)M_w^2} = \frac{1}{(\gamma-1)M_w^2} + 1 \]

Here and in what follows, the pressure \( p \) has been divided by twice the dynamic head at infinity \( \rho_\infty V_w^2 \), the density \( \rho \) by the density at infinity \( \rho_\infty \), and the velocity components \( u, v, \) and \( w \) by the velocity at infinity \( V_w \). On the surface of the body we impose the no-flow condition, and at the bow shock the Hugoniot condition.

For the numerical integration of Eqs. (1) we have developed an algorithm that makes it possible to employ three variants of the Godunov method: a first-order Godunov scheme [7], the Godunov–Kolgan scheme second-order in transverse directions [8], and the Godunov scheme second-order in all directions proposed in [9].

Another distinguishing feature of the algorithm is the use, as in [4], of a marching coordinate \( x \) directed along the edge of the wing (Fig. 1). This makes possible the natural matching of the coordinate systems in zones II and III and the use of difference grids constructed analytically as follows.

At the edge of the wing we use a cylindrical coordinate system moving with the edge; in this case a constant number of cells is preserved with respect to the meridional angle \( \psi \) (Fig. 1) and the step \( \Delta \psi = \text{const} \). When a constant number of cells is preserved in the plane of the wing, the step in zone III will be variable in view of the increasing transverse dimension of the wing. The algorithm is so constructed that \( \Delta \psi = (1-2)\Delta \psi_k \).

When this condition is not satisfied, as many nodes are added to the grid in the plane of the wing as the capacity of the computer will permit. The limitations of the computer mean that when the aspect ratio \( \lambda \geq 100 \) the dimensions \( \Delta \psi \) and \( \Delta \psi_k \) may differ by an order. In this case the use of the Godunov scheme leads to the appearance of considerable oscillations and nonphysical singularities in the distribution of the parameters, as also noted in [6].

The second-order scheme is almost free of this shortcoming, as illustrated by Fig. 1, which shows the shapes of the shock wave in the cross section \( \lambda = 15 \) for flow past a wing with \( \chi = 70^\circ \) when \( \alpha = 0^\circ \) and \( M_\infty = 10 \) obtained in accordance with the two schemes (Godunov (a) and Godunov–Kolgan (b)) using the same difference grids. The calculations made in accordance with the Godunov–Kolgan scheme almost coincide with the data obtained from the second-order scheme, but in view of the fact that in this scheme the step with respect to the marching coordinate can be made almost twice as great and the calculation time shortened by a factor of approximately 1.5, it was used for all the calculations.

Figure 2 shows the pressure distribution (1) in the section \( \lambda = 8 \) for \( \chi = 70^\circ, \alpha = 10^\circ, M_\infty = 6.8, \gamma = 1.4 \), obtained by our method (continuous curve) and the data obtained