Thus, with respect to the effect of the parameters $\varepsilon$ and $\rho$ we see a certain similarity with the lenses described in [6].

The calculations in Table 2 relating to the effect on the shape and size of the lens of the diameter $D$ and the head $h_0$ show that the relation between the quantities $2L$ and $T$ is one of direct proportionality. It is interesting to note that here too the values of $u$ and $v$ hardly vary and are equal to 7.40 and 0.151, respectively.

The authors are grateful to V. M. Entov and A. R. Tsitskishvili for advice and comments.

LITERATURE CITED


STEADY INFLOW INTO A WELL WITH A LONG VERTICAL FRACTURE

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In a departure from previous studies [1, 4] no restrictions are imposed on the shape of the fracture and the effect of the latter on the change in well productivity as a result of hydraulic fracturing and on the lengthwise distribution of the flow through the fracture surface is investigated.

It is proposed to consider the plane steady-state problem of the flow in a reservoir with a vertical hydraulic fracture symmetrical with respect to the well and having a length much greater than the thickness of the reservoir. The fracture is full of sand, whose permeability differs from that of the reservoir. The cross-sectional area of the fracture varies along its length. The flow in the fracture and in the reservoir obeys Darcy's law. The inflow to the walls of the well can be neglected as compared with the inflow to the surfaces of the fracture. The problem can be reduced to a singular integral equation for the distribution of the density of the flow through the fracture surfaces. Its solution is used to estimate the ratio of the well productivity coefficients before and after hydraulic fracturing.

Other methods of estimating the change in well productivity as a result of hydraulic fracturing are described in [1-5]. In [1, 4] plane steady-state problems, in which the fracture is modeled by a region elliptical in plan with a permeability different from that of the reservoir, were considered. In [1] Prats established the existence of a limiting fracture length beyond which there was no further increase in well productivity. This result was then confirmed in [4]. The transient effects associated with inflow into a well with a hydraulic fracture of finite conductivity were investigated in [2-5]. In particular, in [5] the effect of the length and shape of the fracture, defined by the distribution of the sand, on the cumulative output of the well was studied. Some problems of the disturbance of two-dimensional flow by fractures were examined in [6-8].
1. Formulation of the Problem

Let a horizontal reservoir of thickness $2h$ be penetrated by a single well with a vertical symmetrical hydraulic fracture of depth $2H > 2h$ (Fig. 1). The fracture is full of sand, whose permeability $k_f$ differs from that of the reservoir $k$. The expansion of the fracture $2w(x, z)$ is known. The roof and floor of the reservoir are impermeable. The fluid is incompressible, and its viscosity $\mu$ is constant. The fluid pressure in the fracture $p(x)$ is variable along its length, but constant in each vertical section $x = \text{const}$.

The steady flow from the reservoir into the well through the surface of the fracture is described by the following system of equations:

$$
\begin{align*}
\frac{dv(x)}{dx} &= -\frac{k_f}{\mu} \frac{dp(x)}{dx} \\
u(x, y) &= -\frac{k}{\mu} \nabla p_r(x, y) \\
\frac{d}{dx}(Sv) + 4h q_L = 0, & S(x) = 2 \int w(x, z)dz \\
\text{div} u &= 0
\end{align*}
$$

Here, $v$ and $u$ are the permeation velocities in the fracture and in the reservoir, $p_r$ is the pressure in the reservoir, $S(x)$ is the area of the fracture cross section $x = \text{const}$, and $q_L(x)$ is the density of the flow through the surfaces of the fracture in the producing reservoir.

Equations (1.1) and (1.2) are Darcy's law for the motion of the fluid in the fracture and in the reservoir; Eqs. (1.3) and (1.4) are the corresponding flow continuity equations.

Equations (1.1)–(1.4) must be supplemented by the condition of symmetry of the flow in the reservoir, the condition of continuity of the flow through the fracture surface and the condition of continuity of the pressure at the fracture surface

$$
\frac{\partial p_r(x, y)}{\partial y} \bigg|_{y=0} = 0, \quad |x| > L
$$

$$
q_L(x) = u(x, 0), \quad |x| < L
$$

$$
p_r(x, 0) = p(x), \quad |x| < L
$$

We also assume that the rate of flow $q_0$ of the fluid extracted from the reservoir is given. By virtue of the linearity of the problem the quantity $q_0$ does not affect the value of the productivity coefficient of the well in the fractured reservoir and therefore may be chosen arbitrarily. Using the symmetry of the flow about the $y$ axis, we have