THE SHAPE OF A BLOCK OF RESIDUAL VISCOPLASTIC OIL DURING EXPLOITATION OF A CIRCULAR DEPOSIT

V. M. Entov, V. N. Pankov, and S. V. Pan'ko

INTRODUCTION

The task of seeking blocks of residual viscoplastic oil which are in limiting equilibrium in an inhomogeneous stratum reduces in the averaged formulation to solving the equivalent plane problem of nonlinear flow of a homogeneous incompressible fluid in a porous medium [1]. In particular, this scheme reduces for the limiting case of a homogeneous stratum to an effective flow law of the form

\[
\frac{k}{\mu} \nabla p = - \frac{\Phi(w)}{w} w; \quad 0 \leq \Phi(w) \leq \lambda, \quad w = 0; \quad \Phi(w) = \lambda, \quad 0 < w \leq \lambda; \quad \Phi(w) = w, \quad \lambda < w < \infty
\]  

(0.1)

The region \( D_3 \) of zero velocity \( (w = 0) \) corresponds to that part of the stratum where the block occupies the whole thickness; the region \( D_2 \) where the pressure gradient is constant corresponds to water partially washing the stratum; finally, the region \( D_1 \) where the linear resistance law holds, \( \Phi(x) = w \), corresponds to a stratum washed through its whole thickness with only water moving in its cross section.

At the same time, a widely used method for computing blocks of residual oil is a scheme [2, 3] which starts from the a priori assumption that the limiting configuration of the blocks is identical in all the horizontal cross sections of the stratum. It corresponds formally to the discontinuous flow law introduced for the first time in [4]:

\[
0 \leq \Phi(w) \leq \lambda, \quad w = 0; \quad \Phi(w) = w, \quad \lambda < w < \infty
\]  

(0.2)

The corresponding problems are solved in a fairly simple way by the methods of jet theory, but physically meaningful solutions do not exist for all flow configurations. It is shown in [1] that for a number of simple flow schemes the solutions corresponding to the models (0.1) and (0.2) coincide (the region of the partially washed stratum contracts into a line). Those cases in which these schemes lead to other results are of all the more interest. They include the problem, considered in the present study, of the inflow to an eccentrically located well in a circular stratum bounded by a circular supply contour. Within the formulation (0.2), this problem reduces to a nonlinear integrodifferential equation of the same type as Villat’s equation [5, 3], and the configuration of the blocks is determined by solving it numerically for the various different well eccentricities and flow strengths. Not all these solutions are admissible, since the block boundaries constructed have in some cases sections of convexity extending into the flow region, which contradicts the initial physical formulation of the problem.

Within the model (0.1), the problem turns out to be fairly complicated; it is solved in [1] only for a limited range of parameters, but it is analyzed below for the whole range of parameters. It is established that the parameter region is divided into subregions corresponding to qualitatively different regimes; the boundaries of these regimes are found (regime chart), and solutions are constructed for all the characteristic types.

1. Formulation of the Problem. Classification of the Regimes

Within the framework of the averaged formulation already mentioned, this problem reduces to solving the equations of the plane nonlinear flow problem...
for a circle of radius R with center at the origin of coordinates x, y with the point $(-r_0, 0)$, $r_0 < R$, excluded. It is assumed that on the circular contour constant pressure is given, which may be regarded as equal to zero, and at the point $(-r_0, 0)$ the velocity field has a singularity corresponding to a point source (well) of strength $Q$:

$$p = 0 \quad (H = 0), \quad |z| = R; \quad w = (2\pi r^2)^{-1}Q\rho,$$

Here $z$ is the radius vector with origin at the point 0, and $\rho$ is the radius vector with origin at the point $(-r_0, 0)$. For a homogeneous stratum, the effective flow law in a porous medium, $\Phi(w)$, in the system (1.1) is determined by Eqs. (0.1).

After the hodograph transformation, i.e., having taken as independent variables the flow rate modulus $w$ and the angle $\theta$ subtended by the velocity vector with the x axis, we obtain in place of Eq. (1.1) the linear system [1]

$$\frac{\partial \psi}{\partial w} = -\frac{1}{w} \frac{\partial H}{\partial \theta}, \quad \frac{\partial H}{\partial w} = -\frac{1}{w} \frac{\partial \phi}{\partial \theta}, \quad w > \lambda$$

$$\frac{\partial \phi}{\partial w} = 0, \quad \frac{\partial H}{\partial w} = \frac{\lambda}{w^2} \frac{\partial \phi}{\partial \theta}, \quad 0 < w < \lambda$$

Here $\psi$ is the flow function. The hodograph mapping carries the upper half of the flow region (in view of the symmetry it is sufficient to restrict ourselves to considering it) onto the half-strip $\Pi$ with a curvilinear lower boundary $0 \leq \theta \leq \pi$, $w_*(\theta) \leq w < \infty$, where $w_*(\theta)$ is the previously unknown dependence of the flow rate modulus $w$ on the angle $\theta$ along the curvilinear supply contour. The flow chart and the method of constructing it are qualitatively different, depending on the location of the curve $w = w_*(\theta)$ relative to the characteristic lines $w = 0$ and $w = \lambda$ in the plane of the velocity hodograph. In all, five cases are distinguished; they are shown schematically in Fig. 1. One case is trivial, namely, that of very strong flow (a), when $w_*(\theta) > \lambda$ along the whole boundary, the stratum is washed completely everywhere, and the averaged motion is described by the linear flow rate law. The condition for this regime to exist has the form [1]

$$\frac{Q}{2\pi R} \frac{R-r_0}{R+r_0} > \lambda; \quad q \frac{1-e}{1+e} > 1, \quad q = \frac{Q}{2\pi \lambda R}, \quad e = \frac{r_0}{R},$$

The corresponding parameter region in the chart of the regimes (Fig. 2) is denoted by the letter $a$. Here the dimensionless flow strength $q$ and the eccentricity $e$ of the well location are the natural defining parameters of the problem. In the whole of the rest of the region in which the parameters vary,

$$q \leq \frac{1+e}{1-e}, \quad 0 < e \leq 1,$$

there must certainly be a block of residual oil, and the solution is constructed differently for the four qualitatively different cases in Figs. 1b-le.


In all cases there is a region $D_0$ of partly washed stratum in which $0 < w < \lambda$, $|\nabla H| = \lambda = \text{const}$. We have from Eqs. (1.2) and (1.3) the general solution [1] corresponding to the rectilinear streamlines

$$\psi = \psi(\theta), \quad H = \phi(\theta) - \lambda \psi(\theta)/w, \quad z(w, \theta) = z_0(\theta) + w^{-1}e^{\alpha \psi(\theta)} - e^{\theta \psi(\theta)}/w_*(\theta)$$

The functions $\psi(\theta)$, $\phi(\theta)$, and $z_0(\theta)$ in Eqs. (2.1) are determined from the boundary conditions and the conditions for matching to the solutions in other regions.

We can obtain the following equation in the conditions on the supply contour $H = \text{const} = H_0$:

$$\psi(\theta) = H_0 + \lambda \rho(\theta), \quad \phi(\theta) = H_0 + \lambda \psi(\theta)/w_*(\theta)$$