SOLITARY AXISYMMETRIC ROSSBY WAVES OF FINITE AMPLITUDE

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Low-frequency Rossby waves, similar to drift waves in plasma, can propagate in a thin layer of fluid on a rotating sphere. The theory of Rossby waves was developed primarily for quasigeostrophic motions of low amplitude (the relative vorticity is much less than the angular velocity of rotation of the sphere and the layer thickness differs little from the unperturbed thickness) [1]. Of particular interest are nearly axisymmetric solitary waves describing isolated vortex formations moving uniformly opposite to the direction of rotation of the sphere (to the west). Quasigeostrophic waves of low amplitude, inside which the fluid rotates in the opposite direction to the rotation of the sphere (anticyclonically), were considered in [2, 3]. Anticyclones of finite amplitude were investigated in [4], where it was shown that the structure of a solitary wave depends on the previous history of its formation. In this paper, equations are derived for the description of the slow evolution of flows of finite amplitude in a thin spherical layer of homogeneous rotating fluid and the structure of a smooth axisymmetric solitary Rossby wave is analyzed.

The equations of the theory of shallow water on a sphere have the form

\[
R \cos \theta \frac{\partial H}{\partial \tau} + \frac{\partial v_H}{\partial \lambda} + \frac{\partial v_H \cos \theta}{\partial \theta} = 0
\]

\[
\frac{\partial v_h}{\partial \tau} + \frac{1}{R \cos \theta} \frac{\partial B}{\partial \lambda} = \omega_v v_t, \quad \frac{\partial v_s}{\partial \tau} + \frac{\partial B}{\partial \theta} = -\omega_v v_t
\]

\[
\omega = 2\Omega \sin \theta + \frac{1}{R \cos \theta} \left( \frac{\partial v_s}{\partial \lambda} \frac{\partial v_h \cos \theta}{\partial \theta} \right), \quad B = gH + \frac{1}{2}(v_t^2 + v_s^2)
\]
Here, $R$ is the radius of the sphere, $\lambda$ is the longitude, $\theta$ is the latitude, $T$ is the time; $v_x$ and $v_y$ are the zonal and meridional velocity components, $g$ is the acceleration of free fall, and $\omega_z$ is the vertical component of the absolute vorticity.

We consider motions with a horizontal scale $L$ and maximum variation of the thickness of the layer $\Delta h_0$, localized near the latitude $\theta_0$. We determine the scale of the velocity $V$ from the geostrophic balance of forces

$$V = A \frac{L_0 f_0}{L}, \quad L_0 = \frac{\sqrt{g H_0}}{f_0}, \quad f_0 = 2\Omega \sin \theta_0, \quad A = \frac{\Delta h_0}{H_0}$$

where $A$ is the amplitude, and $L_R$ is the Rossby scale. For the time variability scale $T$, we select the time scale of the Rossby wave

$$T = \frac{L}{V_R}, \quad V_R = \frac{L_R^2 f_0}{R \cot \theta_0}$$

where $V_R$ is the maximum phase velocity of the linear Rossby waves.

We introduce the dimensionless variables

$$x = R \lambda \cos \theta_0 / L, \quad y = R(\theta - \theta_0) / L, \quad t = \tau / T, \quad v_x = V_x / V, \quad v_y = V_y / V, \quad h = H / H_0, \quad h' = (h - 1) / A$$

and represent the flow velocity from (1) as follows:

$$v_x = \frac{\partial \psi}{\partial y} - \frac{AM}{Q h} \frac{\partial}{\partial t} \int h' dx, \quad v_y = \frac{\partial \psi}{M \partial x}, \quad M = \frac{\cos \theta}{\cos \theta_0}, \quad Q = \frac{V}{V_R}$$

(4)

Here, $\psi$ is an analog of the flow function. From (1)-(2), it follows that the potential vorticity $\omega_z / H$ is conserved in the moving particle. At the same time, the equation of evolution of the relative anomaly of the potential vorticity $q$ in dimensionless variables takes the form

$$\frac{\partial q}{\partial t} + \frac{Q}{M} J(q) + \frac{\partial \psi}{\partial x} = A \left[ \frac{\partial q}{\partial x} \frac{\partial}{\partial t} \int h' dx - h' \frac{\partial q}{\partial t} \right]$$

(5)

$$J(q) = \frac{\partial v_x}{\partial x} - \frac{\partial}{\partial y} \frac{\partial (M v_x)}{\partial x}, \quad q = \frac{1}{A} \left( \frac{\omega_z h_0}{f_0 H} - I \right) = \frac{B u - I h'}{h}$$

(6)

For slow motions, provided that

$$\frac{1}{L T} = \frac{A B u}{Q} \ll 1$$

we obtain from (2), with allowance for (4), the Bernoulli integral

$$h' + \frac{A B u}{2 h^2} \left[ M^{-1} \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right] = I \psi + A \int q d\psi + c$$

(7)

where $c$ is determined by the boundary conditions of the problem. At the same time, (6) takes the form

$$Bu \left[ M^{-2} \frac{\partial}{\partial x} \left( \frac{\partial \psi}{h \partial x} \right) + M^{-1} \frac{\partial}{\partial y} \left( \frac{M \partial \psi}{h \partial y} \right) \right] = q + (I + A q) h'$$

(8)

The system of equations (5), (7)-(8) for the variables $q$, $h$, and $\psi$ is closed and describes the slow evolution of flows of finite amplitude in a thin spherical layer with allowance for the deviation of the flow velocity from the geostrophic velocity.

For the wave solution which depend on $(x + u t, y)$, the right-hand side of (5) vanishes and we arrive at the equation