\[
\frac{\Gamma^2}{I(\Omega)} = \frac{6}{\pi (1+\gamma^2)^\frac{5}{2}} \frac{1}{Re \sin \theta_0}
\] (2.18)

Since \(I(\Omega)\) is a quantity of the order of \(I_0\), from relation (2.18) it follows that at large Reynolds numbers breakdown of the vortex filament \((Re \sin \theta_0 \gg 1)\) may occur when the relative intensity of the vortex filament is small: \(\Gamma^2/I_0 \ll 1\). In order to determine the mechanism of vortex filament breakdown and the dependence \(\theta_0 = \theta_0(Re)\) as \(Re \to \infty\) it will be necessary to carry out a further investigation of the local flow pattern on longitudinal scales of the order of and smaller than \(\rho_0\).

**LITERATURE CITED**


**FLOW BETWEEN A POROUS ROTATING DISK AND A PLANE**

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It is shown that when a viscous incompressible fluid is sucked through a stationary porous disk spontaneous rotation of the fluid sets in at a certain Reynolds number. This is consistent with the results of a specially designed experiment. Another unusual result is the existence of multicell regimes, corresponding to suction, when the force acting on the porous, rapidly rotating disk is a lift force and, moreover, anomalously large. Charts of the possible steady-state flow regimes, stable and unstable, have been constructed. In the case of fairly intense suction and rotation a stable self-oscillating regime is observed. In the limit of vanishingly small viscosity unusual boundary layer properties associated with suction are noted.

Since Kármán's seminal study [1], in which he examined the self-similar solution of the complete Navier-Stokes equations over a rotating disk, an enormous number of papers have been devoted to the investigation of solutions of this class. Surveys [2, 3] give an excellent account of the history of the question and the present state of research into the problem of Kármán self-similar flow between two impermeable disks. One of the most important results of the analysis is the nonuniqueness of the self-similar solutions of the Navier-Stokes equations. As a rule, the newly generated solutions are characterized by a multicell structure. However, single-cell solutions may also display nonuniqueness. The nonuniqueness of the flow regimes between impermeable disks has been subjected to experimental verification, which showed that for a particular experimental apparatus there exists only one flow regime, although it may be of different types in experiments on different apparatus. In the numerous studies (see [2, 3]) devoted to the numerical investigation of the problem a large number of solutions has been found. The determination of the entire set of solutions for arbitrarily specified angular disk velocities involves considerable computational difficulties. These are a consequence of the fact that the multicell solutions do not all result from bifurcations of a single-cell solution but may be generated as isolated pairs.

In Batchelor's note [4] Kármán-type solutions were extended to flows between infinite rotating disks with given uniform blowing or suction. The presence of additional blowing or suction parameters considerably complicates the problem. The results obtained are reviewed in [5].
In order to make the solution clear and generally amenable to analysis, in what follows we will consider only the problem of flow between a rotating porous disk and a fixed plane. This problem qualitatively models the flow beneath a body supported on an air cushion and is therefore of practical interest. In this case the flow is determined by two parameters: the Reynolds number \( \text{Re} = \frac{Vh}{\nu} \) constructed from the blowing or suction velocity, and the swirl parameter \( K = \frac{\Omega h}{\nu} \), where \( h \) is the distance between disks, and \( \Omega \) is the angular velocity of the porous disk. In relation to a disk on an air cushion with rotation it is more convenient to choose the parameter \( K \) rather than the traditional rotational Reynolds number \( \text{Re}_w = \frac{\Omega h^2}{\nu} \) or the Ekman number \( \text{Ek} = \frac{\text{Re}_w}{\nu} \), since it characterizes only the geometry of the device swirling the flow \([6]\). In general, two more parameters are required: the ratio of the angular velocities of the disks and the ratio of the blowing or suction velocities.

1. Formulation of the Problem

Let the coordinate \( z = 0 \) correspond to a fixed impermeable plane, and the coordinate \( z = h \) to a rotating porous disk through which fluid is uniformly blown or sucked. Seeking the axisymmetric self-similar Kármán-type solution (standard notation) \( v_z = v_z(z, t), v_r = r\omega(z, t), v_r = -(1/2)rv_r' \) and using the Navier-Stokes equations, we obtain the system

\[
\begin{align*}
\frac{1}{\rho} \frac{\partial p}{\partial r} &= -\frac{1}{2} a^2 r \\
\frac{\partial v_z'}{\partial t} &= v_z'''+\frac{1}{2} v_z z - v_z v_z'' - 2\omega^2 - a^2 \delta \\
\frac{\partial \omega}{\partial t} &= v_\omega - v_z \omega' + v_z \omega
\end{align*}
\]

where a prime denotes differentiation with respect to \( z \), \( \delta = \pm 1 \), and \( a \) is an unknown constant which can be eliminated simply by differentiating (1.2)

\[
\frac{\partial v_z'''}{\partial t} = v_z v_z'' - v_z v_z'' - 4\omega \omega'
\]

Upon the system of equations (1.3), (1.4), in addition to the initial conditions, we impose the no-slip boundary conditions

\[
v_z(0) = v_z'(0) = \omega(0) = 0, \quad v_z(h) = V, \quad v_z'(h) = 0, \quad \omega(h) = \Omega
\]

The number of conditions (1.5) corresponds to the order of the system (1.3), (1.4). From the definition of the Reynolds number \( \text{Re} = \frac{Vh}{\nu} \), values of \( \text{Re} > 0 \) correspond to suction, and values of \( \text{Re} < 0 \) to blowing.

Even in the steady-state case the nonlinear boundary-value problem (1.3)-(1.5) is difficult both to investigate and to solve numerically. In order to study the entire set of possible solutions it is expedient to reduce the steady-state boundary-value problem to a Cauchy problem, for which we introduce the new variables

\[
v_z = v_z = \gamma (x, z), \quad v_r = \frac{1}{a} v_r = \alpha (x, z), \quad v_r = v_r = \frac{1}{\sqrt{a}} z
\]

Then the steady-state equations (1.2), (1.3) can be written in a form that does not contain the parameters:

\[
W'' = W'' + \frac{1}{2} W' + 2\gamma^2 + 6; \quad \gamma' = W' - \gamma W'
\]

where a prime denotes differentiation with respect to \( x \). We note that in accordance with (1.1), the quantity \( a \) is expressed in sec\(^{-1}\), so that the variables \( x, W, \) and \( \gamma \) are dimensionless. For system (1.7) we formulate the Cauchy problem

\[
W = 0, \quad W' = 0, \quad W'' = P, \quad \gamma = 0, \quad \gamma' = Q
\]

This contains only the two parameters \( P \) and \( Q \), which also makes it possible to carry out a complete investigation for \( \delta = +1 \) and \( -1 \) separately. By running through the parameters \(-\infty < P < \infty, 0 \leq Q < \infty \) it is possible to obtain the entire family of solutions of