LIFT-TO-DRAG RATIO AT SUPERSONIC SPEEDS

G. I. Maikapar

Wave lift-to-drag ratio is analyzed ignoring friction and using flows behind oblique shock waves and rarefaction waves. It is shown that the lift-to-drag ratio of an infinite oblique plate can surpass considerably that of triangular plates with subsonic, sonic, or supersonic edges. The simplest finite-span oblique wing is a wing with characteristic edges. However, when the normal-to-the-edge flow velocity component behind a shock reaches the speed of sound, the wing contracts into an edge, and other means must be used to exclude the end effect. Several possible variants are indicated. A straight wedge with side plates is the optimal shape for a lifting body with fixed volume, lift, length, and width. Under the same conditions, the cross-section of a pyramidal body formed by stream planes behind one or two plane shocks has practically no effect on the lift-to-drag ratio, while the region of high lift-to-drag ratio is much narrower than for a single wedge. If a pyramid fails to provide the required lift-to-drag ratio, it is necessary to turn to forms that better fill the given area. Redistribution of lift between body and wing permits an improvement in the lift-to-drag ratio.

In order to find the shape of a flight vehicle ensuring the required lift-to-drag ratio on the portion of the flight path under consideration, it is worthwhile estimating the available lift-to-drag ratio and the potential for increasing it. Although the selection of the optimum configuration is determined by numerous other requirements, we shall specify only the volume V, lift Y, length l, and width b. We will consider only the lift-to-drag ratio determined by wave drag, which is most affected by the shape of the body. Of course, the final choice must be made taking friction drag into account, and for convenience we add it to the inverse lift-to-drag ratio. The resulting lift-to-drag ratio is approximately half the ratio for wave drag alone.

All possible aerodynamic shapes fall into the range between a wing and a lifting body. The flows under consideration were modelled using the stream surfaces behind plane shock waves and Prandtl–Meyer rarefaction waves. This approach is preferable to that based on the use of stream surfaces behind oblique cylindrical and axisymmetric shock waves because besides simplifying the calculations, it allows variation of the shapes under consideration by matching the flows behind different shocks. A higher lift-to-drag ratio for stream surfaces behind convex curvilinear shock waves can be achieved by increasing the allowable volume. The schemes under consideration (so-called waveriders) should not be viewed as practical recommendations.

Oblique flow is the major means of reducing the wave drag on a wing [1]. Let the wing be an infinite wedge with the windward and leeward sides inclined at angles δ1 and δ2 in a section normal to the leading edge, which, in turn, lies in the horizontal plane x, z and makes an angle Λ with the freestream velocity Vo directed along the x axis. We assume that the base pressure is zero, which is a conservative assumption and well agrees with the theory [2]. For the lift-to-drag ratio of a wedge the linear theory gives

\[
K_{\text{lt}} = \frac{2c}{\sin \Lambda} + \left( \frac{c}{2} \right) \left( \frac{c}{2C_\sigma} \right), \quad c = \delta_1 - \delta_2, \quad C = \frac{2}{\sqrt{M_0^2 - \sin^2 \Lambda}}, \quad M_0 = \frac{V_0}{a_0}
\]

where \( C \) is the lift coefficient and \( M_0 \) is the Mach number.

We see that for a plate \( c=0, \delta_1=\delta_2=\delta \) with a constant angle \( \Lambda \), the favorable effect of obliqueness decreases with increasing \( M_0 \) [3].

Figure 1 shows the lift-to-drag ratio and the lift coefficient calculated using nonlinear dependences of pressure on angle \( \delta \) for a plane shock wave and a rarefaction wave

\[
K = \frac{C_{\text{lt}}}{\sin \Lambda}, \quad C_{\sigma} = \frac{2 \cos \delta}{\sqrt{M_0^2 - \sin^2 \Lambda}}
\]

where \( \kappa = 1.4 \) is the adiabatic exponent. Points 1 through 8 correspond to \( \Lambda = 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 45^\circ, 60^\circ, \) and \( 90^\circ \), respectively, point 9 to the linear theory, point 10 to [4], point 11 to [5], and point 12 to [6]. Curves a, b, c, and d correspond to \( M_0 = 2, 3, 6, \) and 10, respectively; e, f, and g denote wings with \( M_0 = 2, \Lambda = 45^\circ; M_0 = 3, \Lambda = 30^\circ; \) and \( M_0 = 6, \Lambda = 20^\circ \), respectively. The broken curves correspond to \( M_0 = 2, \Lambda = 25^\circ \) and \( M_0 = 3, \Lambda = 15^\circ \). Finally, the chain curve shows the results obtained using the linear theory \( \Lambda = \sin^{-1}(M_0^{-1}) \) [7], and the dotted curve denotes the limiting value.

A considerable discrepancy between the linear and nonlinear theories is observed for $M_0 = 10$. For smaller Mach numbers the theories yield practically the same results because as compared with the linear theory, an increase in $\delta$ results in a larger increase in pressure behind a shock wave and a smaller increase in rarefaction behind a rarefaction wave.

Figure 1 also shows the lift-to-drag ratio for triangular plates with supersonic leading edges calculated numerically \[4, 5\] and using the approximate theory \[6\], and for plates with sonic and subsonic (taking the suction force into account) edges, calculated using the linear theory \[7\]. According to the numerical solutions \[4\], the lift-to-drag ratio is independent of the angle $\Lambda$. For all $M_0$ the lift-to-drag ratio of an oblique wing exceeds that of a triangular plate owing to the lower load $p_1 - p_2$ on the middle of the latter.

Let us denote the normal-to-the-leading-edge velocity components ahead of and behind a shock/rarefaction wave by $v_1$ and $v_2$, respectively. Then, for the angle $\gamma$ between the velocity vector behind the shock/rarefaction wave and the edge we get

$$\gamma = \arctg \left( \frac{v_2}{v_1} \tan \Lambda \right), \quad v_2 = v_1 \frac{\cos \theta}{\cos (\theta - \delta)}$$

Here, $\theta$ is the angle of inclination of the shock wave in the cross-section normal to the edge, and the flow velocity downstream of the rarefaction wave is tabulated in \[8\].

The angle $\mu$ between a characteristic and the velocity vector behind a shock/rarefaction wave is given by the expression

$$\sin \mu = \sqrt{\frac{M_0^2 - 1/2 (x - 1) \sin^2 \Lambda [1 - (v_2/v_1)^2]}{\cos^2 \Lambda + \sin^2 \Lambda (v_2/v_1)^2}}$$

Figure 1 shows the simplest oblique plate of finite length — a plate with characteristic edges. The angle of the leading end is equal to the smaller of the values $\gamma - \mu$, and that of the rear end to the larger of the values of $\gamma + \mu$, for the windward and leeward sides, thus excluding their influence on the flow behind the shock and rarefaction waves. The smallest angle $\Lambda$ is obtained for $\gamma - \mu = 0$, i.e., for $v_2$ equal to the speed of sound. Then, a finite plate contracts into an edge (in other words, becomes an infinite plate). The dependence of the limiting values of $C_\mu M_0^2$ and $K/M_0$ on $\delta$ is presented in Table 1.\(^*\)

The advantage of oblique wings over triangular ones can be realized by constructing a symmetric wing formed of a pair of centrally connected oblique wings, Fig. 2a. The resulting wing will be the larger, the longer the oblique wings. A sweptforward wing is another possibility, Fig. 2b. In the latter case, the shocks generated by the oblique wings cross and in the plane normal to the line of intersection of the shocks the angle between the shock and the symmetry plane is

\(^*\)According to the linear theory, $K/M_0$ is approximately 10% smaller.