Equations for calculating the limiting equilibrium shapes of the residual oil pillars in homogeneous and stratified inhomogeneous reservoirs of varying wettability are written within the framework of a two-phase displacement model with allowance for capillary pressure. Comparative calculations for an individual well in a circular reservoir show that, in general, the effect of capillary pressure on the shape and volume of the pillars is not negligibly small and may reach 10% of the pore volume.

The limiting equilibrium pillar theory [1–3] is an effective means of estimating the maximum oil yield of flooded reservoirs containing viscoplastic (anomalously viscous) oils. This theory is based on an assumption concerning the immobility of the oil retained in the reservoir by plastic resistance forces. Capillary forces are not considered. Buzinov [4] was evidently the first to take explicitly into account the effect of the capillary forces on the residual saturation distribution of a Newtonian oil, whose immobility was determined by the condition of zero pressure gradient in the oil phase \( |Vp_2| = 0 \). In [5] the capillary well blockage effect was investigated using this condition.

When the residual saturation is determined for a viscoplastic oil with an initial pressure gradient \( G \), the immobility condition takes the form \( |Vp_2| = G \), and the basic system of equations for calculating the limiting water saturation \( s \) with allowance for capillary pressure can be written in the usual notation as follows:

\[
\begin{align*}
\mathbf{div} \mathbf{w}_1 &= 0, \\
|Vp_2| &= G, \\
\mathbf{w}_1 &= -\frac{k}{\mu_1} \mathbf{j}(s) \mathbf{grad} P_1, \\
P_2 &= P_1 + P_3, \\
P_3 &= \cos \theta \frac{t + \mu_1}{k J(s)}
\end{align*}
\]

The residual oil saturation was calculated in this formulation in [6, 7], where it was assumed that the condition of equality of the pressure gradient in the oil to the initial value is satisfied over the entire flow zone.

Below, in continuation of these studies problems of calculating the shape and volume of pillars of residual oil with an initial pressure gradient are formulated for both homogeneous reservoirs and reservoirs with random layerwise inhomogeneity, when the capillary pressure is taken explicitly into account. These formulations are illustrated by the problem of an individual well in the center of a circular field which gives a clear qualitative result for modern regular flooding schemes.

1. The problem of calculating the pillars of residual viscoplastic oil in homogeneous reservoirs can be formulated starting from a model [8] which admits the possibility of three types of domains being formed in the reservoir: completely flushed with water (domains \( D_1 \)), partially flushed (\( D_2 \)), and completely unflushed (\( D_3 \)). Following the same steps as in [8] for a two-phase displacement model with allowance for capillary pressure,

we arrive at the following equations describing the asymptotic pattern of displacement of oil with an initial pressure gradient in a thin homogeneous reservoir:

\[ |\nabla p_2| \geq G(s^*), \quad s = s^*, \quad f_i(s^*) = 0, \quad \text{div} \left( \frac{k}{\mu_i} f_i(s^*) \text{grad } p_i \right) = 0, \quad p_2 - p_i = p_i(s^*), \quad (x, y) \in D_1 \]

\[ |\nabla p_2| = G(s), \quad s_m \leq s \leq s^*, \quad \text{div} \left( \frac{k}{\mu_i} f_i(s) \text{grad } p_i \right) = 0, \quad p_2 - p_i = p_i(s), \quad (x, y) \in D_2 \quad (1.1) \]

\[ |\nabla p_2| \leq G(s), \quad s = s_m, \quad f_i(s) = 0, \quad p_2 - p_i = p_i(s), \quad (x, y) \in D_3 \]

In Eqs. (1.1) x and y are the coordinates of the reservoir strike plane, \( s_m \) and \( s^* \) are the minimum and maximum water saturations, \( p \) is the pressure, \( f \) is the phase permeability, \( G \) is the initial oil pressure gradient, and \( p_3 \) is the capillary pressure; the subscript 1 relates to the water and the subscript 2 to the oil.

In deriving Eqs. (1.1) we made the principal assumption that the limiting water saturation of the reservoir is determined by the absolute value of the pressure gradient in the oil phase. The formulation (1.1) will be consistent if the initial pressure gradient for the oil is assumed to be a monotonically increasing function of the water saturation irrespective of the wettability of the reservoir skeleton. Together with the boundary conditions and the matching conditions on the domain boundaries Eqs. (1.1) constitute a matching boundary-value problem. If, however, in analyzing the experimental data we can construct a function \( G(s) \) such that \( G(s_m) = 0 \) and \( G(s^*) = \omega \), we can forego dividing the flow domain into three subdomains \( D_1, D_2, \) and \( D_3 \) and assume that the domain \( D_1 \) with variable limiting water saturation occupies the entire reservoir. Then instead of a matching problem we arrive at a boundary-value problem, which in general simplifies the calculation of the pillars.

In the absence of an experimental dependence \( G(s) \) we must assume that \( G = \text{const}, \quad s_m \leq s \leq s^* \), and then the domain \( D_1 \) in the formulation (1.1) will be a region of constant pressure gradient and the calculation of the pillars of residual viscoplastic oil in a thin homogeneous reservoir reduces to the solution of the equations for the pressure in the oil phase \( p_2(x, y) \) and the limiting water saturation in domains \( D_1 \) and \( D_2 \) with the matching conditions \( [p_2] = [s] = 0 \) along the boundaries separating domains \( D_1 \) and \( D_2 \) and no-flow conditions along the boundaries of the domains \( D_2 \) and \( D_3 \):

\[ \frac{\partial^2 p_2}{\partial x^2} + \frac{\partial^2 p_2}{\partial y^2} = 0, \quad s(x, y) = s^*, \quad (x, y) \in D_1, \quad \text{div} \left( \frac{k}{\mu_i} f_i(s) \left( \text{grad } p_2 - p_3'(s) \text{grad } s \right) \right) = 0 \]

\[ \left( \frac{\partial p_2}{\partial x} \right)^2 + \left( \frac{\partial p_2}{\partial y} \right)^2 = G^2, \quad (x, y) \in D_2, \quad |\nabla p_2| \leq G, \quad s = s_m, \quad (x, y) \in D_3 \quad (1.2) \]

In formulations (1.1) and (1.2) the limiting water saturation and the associated capillary pressure vary only in the domain \( D_2 \), and in this same domain the action of the capillary forces must be clearly expressed. In those problems in which there is no domain \( D_2 \) the action of the capillary forces will be expressed only through the values of the water saturations \( s_m \) and \( s^* \).

As an illustration of formulation (1.2) we will determine the pillar of residual oil in the case of displacement by water towards a single sink of strength \( Q \) at the center of a plane circular deposit of radius \( R \). For this problem only two domains: \( D_1 \) and \( D_2 \) can exist in the reservoir. In each of the domains we have the relations

\[ \left| \frac{\partial p_2}{\partial r} \right| > G, \quad \frac{Q}{2\pi} = r \frac{k}{\mu_i} f_i(s^*) \frac{\partial p_1}{\partial r}, \quad 0 \leq r \leq r_s, \quad \left| \frac{\partial p_1}{\partial r} \right| = G, \quad \frac{Q}{2\pi} = r \frac{k}{\mu_i} f_i(s) \frac{\partial p_1}{\partial r}, \quad r_s \leq r \leq R \]

from which there follow the equations for the boundary of the pillar and the limiting water saturation within it

\[ r_s \frac{k}{\mu_i} f_i(s^*) G = \frac{Q}{2\pi}, \quad r_r \frac{k}{\mu_i} f_i(s) \left( G - p_3'(s) \frac{ds}{dr} \right) = \frac{Q}{2\pi} \]