A thixotropic fluid with a viscosity dependent on a structural parameter, which satisfies a very simple kinetic equation, is examined. The stability and evolution of shear flows of such a fluid are investigated. Some classes of problems for which approximate solutions can be obtained are considered. Solutions are obtained for problems of changes in structure in oscillatory Couette flows. The apparent viscoelasticity of thixotropic fluids is analyzed. Some aspects of the thixotropic behavior of blood are discussed.

1. The usual meaning of the term thixotropy is a reversible change in viscosity due to the destruction and restoration of the internal, usually supermolecular, structure of the medium under shear. In the case where an increase in shear stresses leads to a reduction of viscosity, the fluid is said to be thixotropic. Thixotropic properties are exhibited by clay solutions, oil products containing impurities, some polymer solutions, and many suspensions, including blood.

We consider a thixotropic fluid whose viscosity \( \eta \) depends on a dimensionless structural parameter \( \lambda \), which satisfies the equation \( \partial \lambda / \partial t = F(\lambda, \varepsilon^{-2} J) \), where \( \varepsilon^{-1} \) is a constant with the dimension of time and \( J \) is the second invariant of the strain tensor. The physical sense of the parameter \( \lambda \) is of no significance in the following treatment (\( \lambda \) is usually the mean elongation of the suspended particles, their numerical concentration, etc.).

Introducing the quantities \( u_*, t_*, h, u_*/h^2, \eta_*, \) and \( 1/q \) as velocity, time, coordinate, pressure difference, viscosity, and function \( F \) scales, respectively, we obtain the following system of equations for developed flow (see Fig. 1) in a region with characteristic diameter \( h \):

\[
S \frac{\partial u}{\partial \tau} = \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p(x, \tau)}{\partial x}, \quad \frac{\partial \lambda}{\partial \tau} = N_1(\lambda, N_1) \]

\[
\tau_{xy} = \eta(\lambda) \frac{\partial u}{\partial y} \quad \tau_{xz} = \eta(\lambda) \frac{\partial u}{\partial z}, \quad I = (\nabla u)^2 \]

\[
S = \frac{\rho h^2}{\eta_1 h^2}, \quad N_1 = \frac{t_*}{q}, \quad N = \varepsilon^{-2} \frac{u_*}{h^2}, \quad t = t_* \tau, \quad F = \frac{f}{q}, \quad J = Iu_*^{-2} / h^2 \]

Here \( \rho \) is the density of the fluid; \( u(y, z, \eta), \tau_{ij}, \) and \( \eta \) are the dimensionless velocity, stress tensor components, and viscosity.

We linearize system (1.1) with respect to the steady flow parameters on the assumption that there are no pressure disturbances. We put

\[
u = u_*(y, z) + \delta u(\tau, y, z), \quad \lambda = \lambda_*(\tau, y, z) + \delta \lambda(\tau, y, z) \]

\[
\eta = \eta_*(y, z) + \delta \eta(\tau, y, z) = \eta_*(\lambda^o) + \eta^o \delta \lambda \]

\[
I = \rho^o + 2 \nu \nabla^2 \delta u \quad (\delta \lambda_1 = 0, \quad \eta^o \delta \eta / \delta \lambda^o) \]

Here the degree sign denotes the steady components, and \( \delta \) denotes the corresponding disturbances.

Substitution of these relations in system (1.1) gives equations for the disturbances:

\[
S \frac{\partial \delta u}{\partial \tau} = \eta^o \Delta \delta u + \delta \eta \Delta \delta u + \nabla^2 \delta u + \nabla \delta \eta \nabla \delta u \]

\[
\delta \eta = \eta^o \delta \lambda, \quad \frac{1}{N_1} \frac{\partial \delta \lambda}{\partial \tau} = f_1 \delta \lambda + 2 f_1 \nu \nabla \delta \lambda \]

\[
f_1 \delta \nu / \delta \eta = \beta \frac{f_1}{\beta^o} \quad (\lambda = \lambda^o, \quad I = I^o) \]

We consider solutions of the form \( \delta u = U \exp i(kr - \omega t) \), \( \delta \lambda = \Lambda \exp i(kr - \omega t) \), assuming that the disturbances propagate in the \((y, z)\) plane against a uniform or slowly varying background; in the last case we assume that the wavelength is small in comparison with \( h(|k| \gg 1) \).

We align the \( y \) axis with the vector \( \mathbf{k} \) (i.e., with the direction of propagation of the disturbances). Then from (1.2), by the usual method, we obtain the dispersion equation in the form

\[
\omega^2 S + \omega \left[ k\eta_\theta^2 + i(k^2\eta^2 - SN_\lambda \xi) \right] + k^2 N_\lambda (\eta^2 \xi^2 - 2\eta_\xi^2 \eta_\theta^2) + ikN_\lambda \left[ 2f_\xi \eta^2 \nabla (\eta_\xi^2 \nabla u^o) - f_\xi \eta_\theta^2 \right] = 0
\]

(1.3)

We first consider the region of large \( k \), for which the disturbances can increase most rapidly. For this we retain in Eq. (1.3) only the main terms, which can affect the behavior of \( \omega \) as \( k \to \infty \) and for any \( N_\lambda \). Introducing the symbol \( s = i\omega \), we obtain

\[
s^2 S - s \left( k^2 \eta_\xi^2 - f_\xi \eta^2 \right) = 0
\]

(1.4)

Hence it follows that as \( k \to \infty \) and when \( N_\lambda \) is finite, the increment \( s \) is limited and, thus, the initial scheme is evolutional. At the same time, \( s \) can take values with a negative real part (of the order of \( N_\lambda \)), which implies the existence of short-wave instability.

To investigate the instability in the whole wavelength range we introduce the symbols:

\[
\chi^2 = \frac{k^2}{S}, \quad \theta = \frac{\eta_\theta^2}{1/S}, \quad \varphi = N_\lambda \xi
\]

\[
F = N_\lambda \left( f_\xi - \frac{2\eta_\xi^2}{\eta^2} \right)
\]

\[
\Phi = N_\lambda \left[ 2f_\theta \nabla (\eta_\xi^2 \nabla u^o) - f_\xi \eta_\theta^2 \right] (S\eta)^{-1}
\]

Equation (1.3) can then be rewritten in the form

\[
\omega^2 + \omega \left( \chi + i(\chi^2 - \varphi) \right) + \chi^2 F + i\chi \Phi = 0
\]

The complex function \( \omega \) has two unique branches, which depend on the real arguments \( \chi, \theta, \varphi \), \( F \), and \( \Phi \). \( \omega, \chi = -\frac{1}{2} \left( \chi + i(\chi^2 - \varphi) \right) \pm \frac{1}{2} \sqrt{D} \),

\[
D = -\frac{1}{4} \left( \chi + i(\chi^2 - \varphi) \right)^2 - \chi^2 F - i\chi \Phi
\]

For \( \sqrt{D} \) here we take the branch which has the value \( \frac{1}{2} \sqrt{|D|} \) when \( \chi = 0 \).

On fulfillment of the condition \( \text{Im} \omega > 0 \), disturbances of the system will increase indefinitely. We have the following regions of instability:

\[
F < 0; \chi^2 < R_-; \quad F \geq 0; \quad \theta^2 - 4F \leq 0; \quad \chi^2 < \infty
\]

\[
F \geq 0; \quad \theta^2 - 4F > 0; \quad \Phi > 0; \quad \chi^2 > R_+; \quad \chi^2 < \infty
\]

\[
F \geq 0; \quad \theta^2 - 4F > 0; \quad \Phi = 0; \quad \chi^2 < \infty
\]

\[
R_\lambda = \left( \theta \Phi \pm \sqrt{\Phi^2 (\theta^2 - 4F)} \right) (2F)^{-1-i\Phi}
\]

It is easy to show that the region of unstable wavelengths becomes greater with increase in \( N_\lambda \) (with reduction of \( \theta \)). The stability condition for all \( \chi \) is satisfaction of the inequalities \( F < 0 \) and \( R_- < 0 \), from which we derive the necessary conditions \( f_\xi < 0, f_\xi - 2\eta_\xi f_\eta \eta_\xi^2 / \eta^2 < 0 \).