COMPUTER CALCULATION OF AERODYNAMIC CHARACTERISTICS OF AIRCRAFT AT SUPERSONIC VELOCITIES

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A previous study by one of the present authors [1] listed a number of works dedicated to calculation of aerodynamic characteristics of aircraft of complex physical construction at supersonic velocities. A method for calculating the flow around a system of small-scale bearing surfaces was developed. The method reduces to determination of the velocity potential $\Phi$ with subsequent differentiation to determine pressure. The present study will present a method of calculating stationary aerodynamic characteristics of aircraft of extensive size at supersonic velocities, in which the basic unknown function is the perturbed pressure $p'$. Eliminating numerical differentiation from the calculation permits an increase in accuracy of the results obtained. The problem is solved for an entire airplane with consideration of the craft's thickness.

1. Formulation of the Problem

In the linear approximation we consider stationary flow around the aircraft by a supersonic flow of ideal gas. The schematization presented in [2] will be used. The pressure drop will be found on planar base elements $S_i$, which are chosen to approximate the corresponding parts of the aircraft surface (Fig. 1). Boundary conditions are transferred from the craft surfaces to the corresponding base elements. In the plane of each element are located a diaphragm $\sigma^i$ and a turbulent shroud $\zeta^i$, while $\sigma'$ is the unperturbed region ahead of the main wave.

We introduce the dimensionless variables

$$
x = x'/b, \quad y = y'/b,
$$
$$
z = z'/b, \quad p = p'/q, \quad q = \Phi/bU_0,
$$

Here $x'$, $y'$, $z'$ is a Cartesian coordinate system (Fig. 1), $b$ is the characteristic linear dimension, $q$ is the velocity head, and $U_0$ is the velocity of the unperturbed flow.

Let the indices 1 and 2 refer to the two sides of $S_i$, while the functions $f_{1,2}(x, y, z)$ describe the deformations of the corresponding sides of the $i$-th element. We introduce the curvature and thickness functions:
\begin{align}
\Delta n'(x, y, z) &= \delta [f_1'(x, y, z) + f_2'(x, y, z)] \\
\Delta n''(x, y, z) &= \delta [f_1'(x, y, z) - f_2'(x, y, z)] 
\end{align}

We write the functions $\varphi$ and $p$ in the form [2]

\begin{align}
q &= \sum_j e_j \varphi_j, \\
p &= \sum_j e_j p_j, \\
e_j &= \{\alpha, \beta, \omega_{\alpha, \beta, z}, \delta, \delta^*\} 
\end{align}

Here $e_j$ are kinematic parameters, $\alpha$ and $\beta$ are attack and slip angles, $\Omega_{x,y,z}$ are projections of the angular velocities, and $\delta$ and $\delta^*$ are deformation parameters in the problem solution, related to curvature and thickness, respectively.

The perturbed pressure and potential are related by the Cauchy–Lagrange integral

\begin{equation}
p_{e_j} = 2 \varphi_{e_j} / \partial x
\end{equation}

The functions $\varphi_{e_j}$ and $p_{e_j}$ then satisfy the wave equation (where $M$ is the Mach number)

\begin{equation}
(1 - M^2) \frac{\partial^2 \varphi_{e_j}}{\partial x^2} + \frac{\partial^2 \varphi_{e_j}}{\partial y^2} + \frac{\partial^2 \varphi_{e_j}}{\partial z^2} = 0
\end{equation}

The boundary conditions on the $i$-th base element may be represented in the form (the positive normal $n^i$ is directed from side 1 to side 2)

\begin{align}
\left( \frac{\partial \varphi_{e_j}}{\partial n^i} \right)_{1,2} &= F_{e_j}'(x, y, z) \\
F_{a,i} = F_{b,i} &= -\cos(n^i, y), \quad F_{c,i} = F_{d,i} = \cos(n^i, z) \\
F_{e,i} &= F_{f,i} = \frac{\partial \Delta n^i}{\partial z}, \quad F_{g,i} = F_{h,i} = \frac{\partial \Delta n^i}{\partial x} 
\end{align}

Conditions on the surfaces $\sigma^i$ and $\Sigma^i$ have the form

\begin{equation}
\Delta \varphi_{e_j} = \hat{\mu}_{e_j, \alpha} - \mu_{e_j, \alpha} = 0
\end{equation}

The Chaplygin–Zhukovskii condition on infrasonic trailing edges ($x^*$ is the edge coordinate) will be

\begin{equation}
\left( \frac{\partial \varphi_{e_j}}{\partial n^i} \right)_{x^*, y, z} = \left( \frac{\partial \varphi_{e_j}}{\partial n^i} \right)_{x, y, z}
\end{equation}