In [1] a model of a wave generator, together with an experimental apparatus to determine the traditional forces generated by the model in water, is described. At the surface of the model six axisymmetric traveling waves are generated, giving rise to motion of the body and the surrounding liquid. The steady flow of liquid caused by oscillations of a cylindrical surface of infinite length was investigated in [2, 3]. The present work investigates the tractional forces of an elongated solid of revolution in a liquid produced by waves traveling over the flexible cylindrical part of the body. The hydrodynamic surface forces are determined by numerical integration of the Navier–Stokes equation. Graphs of the tractional force against the velocity and amplitude of the waves are given.

Consider an elongated body of revolution situated in a viscous incompressible liquid (Fig. 1). Let the flexible cylindrical part of the body surface perform oscillations of the form

\[ r^* = b + \gamma(z, t) \sin(kz - \alpha t) \]  

(1)

This definition of the surface corresponds to moving waves in the direction of the z axis (velocity \( c = \sigma/k \), wavelength \( \lambda = 2\pi/k \), and frequency \( \Omega = \sigma/2\pi \); \( t \) is the time; \( b \) is the radius of the cylinder.

The unsteady axisymmetric flow of liquid caused by the oscillations of the body is described by the equation

\[ \omega = (\omega_0 + \chi_0) + \phi(r, z), \quad \psi = \psi(r, z), \quad \omega_0 = \omega_0(r, z), \quad \omega = \omega(r, z), \quad \omega_0 = \omega_0(r, z) \]  

(2)

where the vorticity \( \omega = u_z - w_r, \) the current function \( \psi, \) and the velocity components \( u \) and \( w \) are related as follows:

\[ \omega = \psi, \quad \omega = \psi, \quad u = \psi, \quad w = -\psi \]  

(3)

The system (2) is integrated with the following initial and boundary conditions:

\[ u = 0, \quad w = 0 \quad (t = 0) \]

(4)

\[ u = u^*, \quad w = 0, \quad (r, z) \in \Gamma_1, \quad u = u^*, \quad w = 0, \quad (r, z) \in \Gamma_1, \Gamma_2 \]

(5)

\[ u = 0, \quad w \rightarrow U(t) \quad (r \rightarrow \infty, \quad z \rightarrow \pm \infty) \]

(6)
Here \( U(t) \) is the velocity of the model; \( \Gamma_1 \) and \( \Gamma_2 \) are the surfaces of the rigid streamlined ends of the solid of revolution; \( \Gamma_3 \) is the deforming surface of the cylindrical part of the model; \( \Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 \).

Note that Eqs. (6) and (5) correspond to forward motion and a fixed body.

The flexural oscillations in Eq. (1) lead to tractional forces, as a result of which the body begins to move with some acceleration \( m\ddot{U}(t) = F(t) \). Simultaneously with traction, drag forces develop at the surface of the body.

The surface force applied to the body from the liquid

\[
F(t) = 2\pi \int_0^L \left[ \left( \frac{p + 2\nu \omega_\pi}{\rho} \right) r_\pi^* + \nu \left( \frac{\partial \omega_\pi}{\partial r_\pi^*} + \frac{\partial \omega_\pi}{\partial \xi_\pi^*} \right) \right] r_\pi^* \, dz
\]

under steady conditions of motion, i.e., \( U(t) \to \text{const} \), is zero.

If the body is fixed, there is no drag, and the force \( F(t) \) is the traction. In Eq. (7), \( m \) and \( L \) are the mass and total length of the body; \( L = L_1 + L_2 + L_3 \); \( L_1 \) and \( L_2 \) are the lengths of the rigid streamlined ends of the body; \( L_3 \) is the length of the cylinder; and \( \nu \) is the kinematic viscosity.

Introducing the dimensionless variables

\[
\begin{align*}
\alpha &= \frac{\omega c}{\omega_\pi}, & \omega &= \frac{\omega c}{\omega}, & r &= \frac{r b}{c}, & z &= \frac{z}{b}, & U &= Uc \\
\psi &= \frac{\psi_0 c}{b}, & \omega &= \frac{\omega c}{b}, & t &= \frac{t b}{c}, & \gamma &= \frac{\gamma_0}{b}
\end{align*}
\]

where \( \gamma \) is the relative amplitude of the traveling wave, and \( b \) is the radius of the cylindrical part of the model, Eqs. (2)-(6) retain their previous form, except that \( \nu \) is replaced by the dimensionless variable \( 1/Re \), where \( Re = \frac{cb}{\nu} \); below, the bar above the dimensionless quantities will be omitted.

The system of differential equations (2), (3) with the conditions (4)-(6) are solved by the finite-difference method. The principle of construction of the rectangular calculational grid with nonuniform step is shown in Fig. 1. The standard Taylor-series expansions for the first and second derivatives are

\[
\begin{align*}
\frac{\delta f}{\delta r_{i,j}} &= \frac{\Delta r_{i-1,j} f_{i+1,j} - \Delta r_{i+1,j} f_{i,j}}{2r_{i,j}}, \\
\frac{\delta^2 f}{\delta r_{i,j}^2} &= \frac{\Delta r_{i} (r_{i+1,j} f_{i+1,j} - r_{i+1,j} f_{i,j})}{2r_{i,j}}, \\
\frac{\delta^2 f}{\delta z_{i,j}^2} &= \frac{\Delta r_{i+1,j} f_{i+1,j} - 2f_{i,j} + \Delta r_{i} f_{i-1,j}}{2r_{i,j}}.
\end{align*}
\]

Here and below \( i \) is a subscript increasing in the direction of the radial coordinate \( r \); \( j \) is a subscript increasing in the direction of the \( z \) axis; and \( \Delta r_{i} \) and \( \Delta z_{j} \) are the steps along \( r \) and \( z \), respectively. The approximation of the derivatives \( \delta f/\delta z_{i,j} \) and \( \delta^2 f/\delta z_{i,j}^2 \) is as in Eq. (8).

The nonlinear terms in Eq. (2) are approximated by differences taken in the direction opposite to the flow [4, 5]:

\[
\frac{\delta (u\omega)}{\delta r_{i,j}} = \begin{cases} 
\frac{(u\omega)_{i+1,j} - (u\omega)_{i,j}}{\Delta r_{i,j}}, & u_{i,j} < 0 \\
\frac{(u\omega)_{i,j} - (u\omega)_{i-1,j}}{\Delta r_{i,j}}, & u_{i,j} > 0
\end{cases}
\]

If the sign of \( u_{i,j} \) changes close to the grid point, the calculation scheme is transformed so that the conservation law is satisfied [6]. An expression analogous to Eq. (9) is used to approximate the term \( \delta (u\omega)/\delta z_{i,j} \).

The equation of parabolic type (2) is solved by an explicit two-layer scheme. At each time step, the values of \( \psi_{i,j} \) are found from the equation of elliptical type (3) by the iterative scheme

\[
\psi_{i,j} = \psi_{i,j} + \beta_{i,j} \left( \frac{\delta^2 \psi_{i,j}}{\delta z_{i,j}^2} + \frac{\delta^2 \psi_{i,j}}{\delta r_{i,j}^2} - \frac{\delta \psi_{i,j}}{\delta r_{i,j}} r_{i,j} \right)
\]