error of 10%, the error in determining $b_i$ and $w$ is approximately 100% at $f = 10^6$ Hz.

Thus, by means of relations (15) and (16) for a known temperature and recombination coefficient it is possible to determine the ion mobility and the effective ionization rate in the equilibrium plasma. For this purpose it is necessary to measure the probe current oscillation amplitude and the phase shift between the current and potential oscillations at a selected frequency or to measure the amplitude of the current oscillations at two different frequencies. The undisturbed equilibrium electron concentration $n_0$ is then found from the relation $n_0 = \left(\frac{w}{\alpha}\right)^{1/2}$.

LITERATURE CITED


INVESTIGATION OF THE FLOW PAST WINGS OF INFINITE SPAN WITH A BLUNT LEADING EDGE IN THE VORTICITY INTERACTION REGIME

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An asymptotic analysis of the Navier–Stokes equations is carried out for the case of hypersonic flow past wings of infinite span with a blunt leading edge when $\epsilon \to 0$, $Re \to \infty$, and $M_\infty \to \infty$. Analytic solutions are obtained for an inviscid shock layer and inviscid boundary layer. The results of a numerical solution of the problems of vorticity interaction at the blunt edge and on the lateral surface of the wing are presented. These solutions are compared with the solution of the equations of a thin viscous shock layer and on the basis of this comparison the boundaries of the asymptotic regions are estimated.

The vorticity interaction regime in the neighborhood of a blunt surface has previously been examined in [1, 2] for axisymmetric bodies and in [3] for plane bodies. Lateral surface vorticity interaction in the axisymmetric and three-dimensional cases was examined in [4–8].

1. Formulation of the Problem and Classification of Flow Regimes

We will examine a wing of infinite span in a hypersonic viscous heat-conducting gas flow in the presence of a slip angle. We will consider bodies for which the Newtonian
theory of flow past blunt bodies does not give separation of the shock layer. In par-
ticular, these bodies will include cylinders with hyperbolic contours characterized by
the radius of curvature of the contour at its vertex $R$, the length of the contour $L$
reckoned from the vertex, and the asymptote half-angle $\beta$, although the results obtained
are also applicable to bodies of other shapes. We begin with the system of Navier-
Stokes equations written in modified Mises variables

\begin{align}
D_u + u &= -\frac{2e\alpha}{\rho u(1+\varepsilon)} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \frac{l}{K} \frac{\partial u}{\partial \Psi} \right) + \ldots, \quad D_w = \frac{\partial}{\partial x} \left( \frac{l}{K} \frac{\partial w}{\partial \Psi} \right) + \ldots, \quad \frac{2e}{(1+\varepsilon)} \frac{\partial p}{\partial \Psi} = a_x x^2 u + \ldots,
\end{align}

\begin{align}
D_T &= \frac{2e}{(1+\varepsilon)c_x} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \frac{l}{K} \frac{\partial T}{\partial \Psi} \right) + \ldots, \quad \frac{2e}{(1+\varepsilon)c_x} \frac{\partial p}{\partial x} = \frac{\partial}{\partial \Psi} \left( \frac{\partial w}{\partial \Psi} \right)^2 + \ldots,
\end{align}

\begin{align}
\frac{\partial^2 \psi}{(1+\varepsilon)\rho} = \frac{1}{\rho\psi}, \quad v = au D_y, \quad \rho = \rho T, \quad \mu = \tau_c, \quad D_x = x \frac{\partial}{\partial x} - \frac{\partial}{\partial \psi}, \quad \alpha = 1 + \varepsilon \psi.
\end{align}

Here, $Rx$ is the coordinate reckoned from the stagnation point along the directrix,
$Ry$ the coordinate reckoned along the generator, and $e\alpha$ the coordinate reckoned along
the normal to the surface of the body; $xu_u, \varepsilon u_v$, and $w_\alpha w_\nu$ are the components of the ve-
locity vector; $U_u = V_x \cos \varphi$, $W_u = V_y \sin \varphi$, where $V_m$ is the modulus of the total approach
stream velocity, and $\varphi$ is the slip angle; $\varepsilon^{-1}\rho_\omega$ is the density, $(\varepsilon\rho)^{-1}\rho_\omega$ the pressure,
$\delta^{-1}\rho_\omega$ the viscosity, and $\delta^{-1}\rho_\omega T$ the temperature of the gas; $\rho_\omega u_\alpha u_\psi$ is the stream func-
tion introduced in accordance with the expression (see [9]) $d\psi = -\rho v_{\alpha-1} dx + \rho u_{\psi} dy$.
Moreover, we have introduced the following notation:

\begin{align}
\psi &= \frac{x_0}{x}, \quad \varphi = \frac{x_0 - 1}{\gamma + 1}, \quad l = \mu\psi, \quad \psi = \frac{c_p}{c_t}, \quad \delta^{-1} = (\gamma - 1) M_m^{-2}, \quad M_m^2 = U_m^{-2}(\frac{\rho_m}{\rho_m})^{-1}.
\end{align}

In these variables the surface of the body corresponds to $\psi = 0$ ($\psi_0 = 0$), the
shock wave to $\psi = \psi_S = r_S x^{-1} = x^{-1}(r_w + \varepsilon \psi_y \cos \alpha)$, where $r_w(x)$ is the distance from the
plane of symmetry to the surface of the wing, and $\alpha(x)$ is the angle between the vector $U_m$
and tangent to the wing contour.

For convenience, we introduce the variable $z = \exp(y)$. Then the fifth equation of
the system (1.1) takes the form:

\begin{align}
\frac{\partial z}{\partial \psi} = \frac{z}{\rho \psi}.
\end{align}

The boundary conditions at the surface of the body and in the free stream are as
follows:

\begin{align}
u = -1, \quad T = T(x), \quad y = 0, \quad (z = 1) \quad (\psi = 0)
\end{align}

\begin{align}
u = \frac{\cos \alpha}{x}, \quad v = \frac{\sin \alpha}{\varepsilon}, \quad u = 1, \quad (\psi = \infty)
\end{align}

In what follows we assume that $\varepsilon \rightarrow 0$, $\delta \rightarrow 0$, and $Re \rightarrow \infty$. In the asymptotic solu-
tion the flow region between the body and the shock wave is divided into series of sub-
regions, the number of which will depend on the relationship among the determining par-
parameters of the problem. We will consider three regimes.

1. $K = \varepsilon$ $Re = O(1)$, $N = \varepsilon KL^{-1} \gg 1$. The flow is divided into three regions, when
$x < O(K^{-1})$ we get a viscous shock layer in which $\psi_\alpha = O(K^{-1})$, and when $x \gg O(K^{-1})$ an in-
viscid shock layer ($\psi_\alpha = O(\infty)$). In the neighborhood of the blunt edge we get a vorticity interac-
tion regime, and the entire flow region between the shock wave and the body is divided into four regions. Near the shock wave there is an inviscid shock layer ($\psi_\alpha = O(\infty)$). Near the body when $x \ll O(K)$ we get a boundary layer with vorticity inter-
taction ($\psi_\alpha = O(K^{-1})$), when $x = O(K)$ a region of entropy layer absorption ($\psi_\alpha = O(1)$),
and when $x \gg O(K)$ a viscous boundary layer ($\psi_\alpha = O(\varepsilon K^{-1})$).

2. $N = Re \varepsilon n = O(1)$, $1 < n \leq 1.5$, $M = KL^{-1} \ll 1$. In the neighborhood of the blunt
edge we get a vorticity interaction regime, and the entire flow region between the shock wave
and the body is divided into four regions. Near the shock wave there is an inviscid shock layer ($\psi_\alpha = O(\infty)$). Near the body when $x \ll O(K)$ we get a boundary layer with vorticity inter-
taction ($\psi_\alpha = O(K^{-1})$), when $x = O(K)$ a region of entropy layer absorption ($\psi_\alpha = O(1)$),
and when $x \gg O(K)$ a viscous boundary layer ($\psi_\alpha = O(\varepsilon K^{-1})$).