USE OF THE AERODYNAMIC EQUIVALENCE METHOD IN THE DETERMINATION
AND ANALYSIS OF THE AERODYNAMIC COEFFICIENTS OF ASYMMETRIC BODIES

G. G. Skiba and A. N. Tsar'kov

In [1] the validity of the linear method of aerodynamic equivalence (AE) was demonstrated and the results of calculating the aerodynamic characteristics (ADC) of certain asymmetric (nonaxisymmetric) bodies, differing only slightly from the axisymmetric, were presented. In this paper a nonlinear AE method is proposed. This method is based on the principle of the equivalence of two bodies, one of which has a cross section of arbitrary shape while the other has a cross section described by a smooth function. This function is the sum of the first \( N + 1 \) terms of the Fourier series of the initial (discontinuous) function describing the shape of the body. The effectiveness of the AE method is illustrated with reference to certain examples of star-shaped bodies. The accuracy of the results obtained is estimated and a comparison is made with the experimental data. It is also shown that the AE method makes it possible to give a simple explanation of certain results of aerodynamics from a new standpoint.

1. Statement of the Problem

It is proposed to consider the three-dimensional motion of an asymmetric body in an inviscid nonheat-conducting gas. We introduce the right-handed Cartesian coordinate system \( XYZ \) (see Fig. 1), which corresponds to the cylindrical coordinate system \( XR\phi \) used in the numerical integration of the gas dynamic system of equations.

The motion of the gas relative to the body is described by the three-dimensional stationary system of equations

\[
\frac{\partial}{\partial t} (\rho V) + \nabla \cdot (\rho V V) = 0, \quad \frac{\partial}{\partial t} (\rho e) + \nabla \cdot (\rho e V) = 0, \quad \frac{\partial}{\partial t} (\rho a^2) + \nabla \cdot (\rho a^2 V) = 0, \quad i = 1, 2, \ldots, \quad \rho = \rho(p, \varphi), \quad a^2 = a^2(p, \varphi)
\]

where \( V, \rho, a, \) and \( i \) are the velocity vector, pressure, density, speed of sound, and enthalpy of the gas, respectively. The free-stream parameters are denoted by the subscript \( \infty \).

The boundary condition at the surface of the body, defined by the known function \( R = G_t(X, \varphi) \), is written as follows:

\[
V \nabla G_t = 0
\]

(1.2)

The function \( G_t(X, \varphi) \) is determined in the process of solving the problem.

The boundary conditions at the surface of the bow shock wave, defined by the known function \( R = R_b(X, \varphi) \), are represented in the form:

\[
(V + V_s) \tau = 0, \quad (V + V_s) r = 0, \quad \rho_s(V_n) = -p(V_n) \]

\[
p_s + \rho_s(V_n)^2 = p + p(V_n)^2, \quad (V_n)^2 + 2 + i = (V_n)^2 + 2 + i
\]

(1.3)

Here, $V_e = V_0$ is the translational velocity vector; $\tau_1$ and $\tau_2$ are mutually perpendicular vectors in the plane tangential to the wave surface; $n = V_R / |V_R|$ is the outward unit normal to the surface of the bow shock.

In system of equations (1.1) and boundary conditions (1.2), (1.3) the parameters $V$, $p$, $\rho$, $i$, and $a$ are represented in dimensionless form.

2. Method of Solution

The function $R = R_\tau(X, \varphi)$ describing the shape of the surface of the asymmetric body is assumed to be known. As a result of integrating system of equations (1.1) with boundary conditions (1.2), (1.3) it is possible to determine the pressure distribution on the surface of the body $p(X, \varphi)$ and the corresponding aerodynamic coefficients.

In this paper the problem is solved on the basis of the AE method. In the AE method the bounded (in the general case discontinuous) functions $p(X, \varphi)$ and $R_\tau(X, \varphi)$ periodic in $\varphi$, expanded in Fourier series and substituted in the integral relations for the aerodynamic coefficients, lead to a conclusion concerning the existence of another asymmetric, but smooth body. This smooth body is equivalent to the initial body inasmuch as the corresponding aerodynamic coefficients are equal to within a certain predetermined degree of accuracy (within the framework of the mathematical model).

Without loss of generality, we assume that the function $R_\tau(X, \varphi)$ is even in $\varphi$. Representing it by a Fourier series, we obtain

$$R_\tau(X, \varphi) = \sum_{n=0}^{\infty} b_n \cos(n\varphi), \quad b_n = \frac{1}{n} \int_0^\pi R_\tau(X, \varphi) \cos(n\varphi) d\varphi$$

$$R_\tau(X, \varphi) = \frac{2}{n\pi} \int_0^\pi R_\tau(X, \varphi) \cos(n\varphi) d\varphi$$

(2.1)

The general integral expressions for the aerodynamic coefficients (see, for example, [1]) contain the values of the derivatives $\partial R_\tau / \partial X$ and $\partial R_\tau / \partial \varphi$. It is assumed that these values are determined from the following relations:

$$\frac{\partial R_\tau(X, \varphi)}{\partial X} \approx \sum_{n=0}^{N} \frac{db_n}{dX} \cos(n\varphi), \quad \frac{\partial R_\tau(X, \varphi)}{\partial \varphi} \approx -\sum_{n=0}^{N} \left[ nb_n \sin(n\varphi) \right]$$

(2.2)

We will illustrate the basic principles of the AE method with reference to the calculation of the normal force coefficient:

$$C_n = \frac{1}{qS} \int_0^{2\pi} \left( R_\tau \cos \varphi + \frac{\partial R_\tau}{\partial \varphi} \sin \varphi \right) dX d\varphi$$

(2.3)