SOUND PROPAGATION IN A PLANE WAVE GUIDE WITH AN ELASTIC WALL SECTION

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Problems of acoustic wave propagation in a plane wave guide whose walls are assumed to be undeformed with the exception of a section of finite length whose bending is described by the thin plate theory equations in the framework of the Kirchhoff–Love hypotheses are considered. The soundproofing characteristics of the wave guide described and the stability of the forced oscillations of the system considered are investigated. Formulations of the problem of active vibroacoustic protection and the problem for the peristaltic pump are given. Soundproofing in wave guides has been considered in a number of papers, a fairly complete review of which is given in [1].

1. We consider the problem of generating acoustic waves in a plane wave guide (Fig. 1) by means of the time-periodic load \( q(x, t) = Qe^{-i(\omega t + \xi t)} \) applied to the elastic section of the wall. The equations of the Kirchhoff–Love thin plate theory are used to describe the deformation of the elastic element of the wave guide wall. It is assumed that the wave guide is filled by a perfect compressible fluid, in which the wave processes will be described by the acoustic approximation equations.

Eliminating the time variable and assuming that the time dependence of all the unknown quantities is expressed by the factor \( e^{-i\omega t} \) in the coordinate system shown in Fig. 1, we obtain the following set of equations:

\[
\Delta \varphi + \frac{\omega^2}{c^2} \varphi = 0, \quad x \in (-\infty, +\infty), \quad y \in (0, H) \tag{1.1}
\]

\[
y = 0: \quad \frac{\partial \varphi}{\partial y} = 0, \quad x \in (-\infty, +\infty), \quad y = H: \quad \frac{\partial \varphi}{\partial y} = 0, \quad |x| > a \tag{1.2}
\]

\[
\frac{\partial \varphi}{\partial y} = -i\omega w', \quad Dw'' - \rho_0 h w = i\rho_0 \omega \varphi - Qe^{-i\xi t}, \quad |x| < a, \quad w(\pm a) = w' (\pm a) = 0 \tag{1.3}
\]

Here \( \varphi \) is the acoustic potential, \( \rho_0, c \) are, respectively, the density of the fluid and the velocity of sound in it, \( w \) is the vibrational mode of the plate, \( D, h \) are the cylindrical rigidity and the thickness of the plate, \( \rho \) is the material density of the plate, \( H \) is the distance between the rigid walls of the wave guide, and \( a \) is the half-length of the elastic insertion. We note that Eqs. (1.3) describe the conditions
of attaching the plate in the rigid walls of the wave guide.

To close the system of equations (1.1)-(1.3) the radiation conditions [2] must be added.

In the regions \(|x| > a\), we will find the acoustic field generated by the external load in the form of series in natural outgoing waves of the wave guide with rigid walls [3], namely,

\[
\varphi(x, y) = \sum_{n=0}^{\infty} A_n z_n e^{\pm in(y/\rho_H)} \cos \left( \frac{\pi n y}{H} \right), \quad |x| > a
\]

\[z_n = \left( \frac{\omega^2 - \frac{\pi^2 n^2}{H^2}}{c^2} \right)^{1/2}, \quad n \leq \frac{\omega H}{\pi c}; \quad z_n = \left( \frac{\frac{\pi^2 n^2}{H^2} - \frac{\omega^2}{c^2}}{\pi c} \right)^{1/2}, \quad n > \frac{\omega H}{\pi c}
\]

We introduce into the discussion the meromorphic function \(L(u)\) of the complex variable \(u\), such that \(n\) are its poles, and the equation

\[
(D^2 - \frac{u^2}{c^2}) L(u) = \frac{\gamma(u)}{\gamma(u)}, \quad L(\alpha) = D\alpha', \quad \gamma(u) = \left( \frac{u^2 - \frac{\omega^2}{c^2}}{\pi c} \right)^{1/2}
\]

Equation (1.5) is the dispersion relation for an infinite acoustic wave guide, of which one wall is rigid, while the other is elastic, modeled by the Kirchhoff-Love plate [4].

For any real \(\omega\), Eq. (1.5) has a finite (nonzero) number of real roots corresponding to the freely propagating (undamped) waves. All the roots of Eq. (1.5) lying either on the positive side of the real axis or in the upper half-plane of the complex variable \(u\) correspond to waves propagating to the right; we denote this set of roots \(\Omega_+\). Similarly, the set \(\Omega_-\) corresponds to the waves propagating to the left; if \(\zeta \in \Omega_+\), then \(-\zeta \in \Omega_-\), and vice versa.

Using the Rouché theorem [5], it can be shown that for sufficiently large \(n\) in the band \(0 \leq \text{Im} \ u \leq \pi(n + \frac{1}{2})/H\) there are exactly \(n + 3\) elements of the set \(\Omega_+\), whereas the same band contains only \(n + 1\) of the points \(z_n\) (\(n = 0, 1, 2, \ldots\)). We number all the points of the set \(\Omega_+\) in ascending order of the moduli and denote them \(\zeta_m\) (\(m = -2, -1, 0, 1, \ldots\)). We have the asymptotic relation

\[
|\zeta_n - z_n| = O(n^{-2}), \quad n \to \infty
\]

We represent the deflection \(w\) and the potential \(\varphi\) in the region \(|x| < a\) in the form

\[
\varphi(x, y) = -i\omega \left[ W_e^{-i\nu \psi(\eta, y)} + \sum_{m=-2}^{\infty} W_m(x) \psi(\zeta_m, y) \right]
\]

\[
w(x) = W_e^{-i\nu \psi(\eta, y)} + \sum_{m=-2}^{\infty} W_m(x), \quad W_m(x) = W_m^+ e^{-i\eta_m(x-a)} + W_m^- e^{i\eta_m(x+a)}, \quad \psi(\eta, y) = \frac{\text{ch}[\gamma(u)y]}{\gamma(u) \text{sh}[\gamma(u)y]}
\]

It can easily be shown that the solution in the form (1.4), (1.7) satisfies all the equations (1.2) provided \(L(\eta)W_0 = -Q\). Here and below, unless otherwise stated, it is assumed that \(\eta \neq \pm z_n, \eta \neq \pm \zeta_m\) (\(n = 0, 1, 2, \ldots, m = -2, -1, 0, 1, \ldots\)) and that all the zeros \(\zeta_m\) and the poles \(z_n\) of the function \(L(u)\) are simple.

The solutions (1.4) and (1.7) satisfy Eq. (1.1) only in the corresponding regions