In the context of the problem of describing the transition of a laminar boundary layer to a turbulent, great interest attaches to the study of susceptibility, i.e., of the reaction of the flow to various external influences, such as acoustic perturbations, surface roughness, vibration of the wall, turbulence of the unperturbed flow, etc. A general property of the effect of the factors mentioned above on the flow in a laminar boundary layer was discovered in experimental and numerical studies and is noted in [1]: in all cases an external forcing perturbation leads to the excitation of normal modes of oscillation in the boundary layer which propagate downstream, namely, Tollmien-Schlichting waves. There is an analytical calculation in [2, 3] of the amplitude of a wave excited by harmonic oscillations of a narrow band on the surface of a plane plate, the Reynolds number having been assumed to be infinitely large, and the frequency of the vibrator corresponding to the neighborhood of the lower branch of the neutral curve [4]. In [5] the amplitude of the wave of instability generated is calculated by the method of expansion of the solution in a biorthogonal system of eigenfunctions. The amplitudes of the Tollmien-Schlichting waves are calculated below by means of a generalization of the method of [2] for the whole range of Reynolds numbers and frequencies of the vibrator corresponding to the region of instability: for moderate Reynolds numbers the problem is solved numerically, while for large Reynolds numbers an asymptotic solution is constructed.

1. Formulation of the Problem

Let us consider the excitation of small oscillations in steady flow of a viscous incompressible fluid around a plane semi-infinite plate. We shall assume that the free stream is uniform and parallel to the plane of the plate. As basic units of measurement we shall use the density \( \rho \) of the fluid, the velocity \( U_\infty \) of the free stream, and the characteristic length \( l = \lambda U_\infty / \tau \) (\( \lambda \) is the coefficient of viscosity, and \( \tau \) is the tangential stress at the point \( O \) on the surface of the plate). The Reynolds number is given by \( R = \rho U_\infty l / \lambda \gg 1 \).

We introduce a Cartesian system of coordinates with origin at the point \( O \), \( x \) axis
directed along the velocity of the free stream, and y axis perpendicular to the surface around which flow is taking place (Fig. 1). We shall denote the profile of the velocity of the unperturbed flow at the point O by \( U(y) \).

We shall assume that the perturbations of the main flow are due to vibration of a section of the surface of the plate whose points are displaced in the vertical direction in accordance with the law

\[
y(x) = \sigma f(x) \exp(-i\omega t)
\]

(1.1)

Here \( t \) is the time, \( \omega \) is the frequency, \( \sigma \ll 1 \) is the amplitude parameter; the function \( f \) is identically equal to zero outside a certain segment located in the neighborhood of the point O, and the length of this segment is much less than the distance \( L \) to the leading edge. We shall solve the problem under the assumption that the main flow is parallel, and so we shall assume in what follows that \( |x| \ll L \) (although it is possible for \( |x| \gg 1 \) to hold).

The perturbations of the parameters of the flow (the components of the velocity vector and the pressure) will be denoted by \( \varphi \) (\( \varphi = v_x, v_y, p \)). The functions \( \varphi \) satisfy the linearized system of Navier–Stokes equations. On the assumptions made, the coefficients of the system depend only on the variable \( y \), and so we shall seek a solution to the problem by means of a Fourier transformation in \( x \).

\[
\varphi = \int_{-\infty}^{\infty} f^*(k) \tilde{q}^*(k, y) \exp(ikx-\omega t) \, dk
\]

Here the Fourier transforms of the unknown functions are normalized by \( \tilde{f}^* \) for convenience. The transformation (1.2) makes it possible to reduce the system of partial differential equations for the perturbations \( \varphi \) to a system of ordinary differential equations for their Fourier components.

We introduce the constant \( c = \omega/k \), and also the new unknown function \( \varphi \) by putting \( \varphi^* = -ik\varphi \). The problem of calculating \( \varphi^* \) reduces as a result to the solution of the Orr–Sommerfeld equations with the inhomogeneous boundary conditions

\[
(U-c) \left( \frac{d^2\varphi}{dy^2} - k^2\varphi \right) - \frac{d^2U}{dy^2} \varphi = \frac{1}{ikR} \left( \frac{d^2\varphi}{dy^2} - 2k^2 \frac{d\varphi}{dy} + k^2\varphi \right)
\]

(1.3)

\[
\varphi(0) = C, \quad \frac{d\varphi}{dy}(0) = -1, \quad \varphi(\infty) = 0
\]

The first two boundary conditions in (1.3) follow from no-slip conditions for the wall, namely, (1.1). The last condition in (1.3) expresses the requirement that the perturbations should damp as \( y \to \infty \); it excludes two exponentially increasing linearly independent solutions of Eq. (1.3).

We shall investigate the solutions (1.2) and (1.3) as \( x \to \infty \). As in [2], we continue \( \varphi^* \) into the region of complex \( k \) and replace the first integral in (1.2) by an expression consisting of two terms: an integral over the contour \( \Gamma \) lying in the upper half-plane, and the sum, multiplied by \( 2\pi i \), of the residues at the poles occurring in the region between the real axis and \( \Gamma \) (the first-order poles of the functions \( \varphi^* \) are the eigenvalues of the homogeneous problem for Eq. (1.3) arising in the study of the stability of the flow in the boundary layer [6]). If the parameters \( \omega, R \) correspond to the