The quantity \( \frac{U_0^2}{D_2} \) must be considered as interference. If we assume that both stars radiate in practically the same manner (\( U_0^2 \approx U_0^2 \)), then the deleterious effect of the interference may be neglected under the condition that

\[
\frac{D_0^2}{(D_1 + D_2)^2} q(D_p, x_p) \gg 1.
\]

This inequality is easily realized for close-lying star pairs, removed a significant distance from the observer (\( D_p \gg D_0 \)). In this case the old condition \( q \gg 1 \) proves to be sufficient for detection of the focusing effect.

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LITERATURE CITED


THE KINETIC EQUATION METHOD FOR ACOUSTIC-GRAVITY WAVES

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The dispersion equation for acoustic-gravitational waves is obtained using the method of the kinetic equation. The damping decrement of these waves produced by viscosity and thermal conductivity is calculated for vertical propagation in an isothermal atmosphere.

In theoretical studies of radio propagation questions and in the practical establishment of terrestrial and space communications, one must consider the presence of inhomogeneities of various sizes. It is known that coarse scale perturbations of the wave type recorded in the atmosphere from the surface of the earth to heights exceeding the F-layer maximum are related to propagation of acoustic-gravitational waves.

At the present time, the question of peculiarities in propagation of acoustic-gravitational waves in an inhomogeneous atmosphere has been studied in minute detail (see, e.g., [1-4]). The solutions obtained employed macroscopic hydrodynamics equations, which can be considered inapplicable at high altitudes due to the significant increase in free path length \( l_f \).

At the same time, for sound and plasma waves, the kinetic equation method [5, 6] has been employed in addition to the hydrodynamic description. In the present study the kinetic equation method will be adapted for use in an inhomogeneous medium and used to derive the dispersion equation of acoustic-gravitational waves and to calculate their damping by viscosity and thermal conductivity.

The kinetic equation for the distribution function \( f(t, r, \xi) \) of atmospheric particles with velocities \( \xi \) in a space with Cartesian coordinates \( r(x, y, z) \) is written in the form

\[
\frac{\partial f}{\partial t} + \nabla \cdot (\xi f) - \frac{1}{\rho} \nabla \cdot (\rho \xi f) = \frac{\partial}{\partial \xi} \left( \frac{\partial f}{\partial \xi} \right) - \frac{1}{\rho} \left( \frac{\partial}{\partial \xi} \rho \frac{\partial f}{\partial \xi} \right).
\]

Here $g = (0, 0, -g)$ is the acceleration of gravity, which will now be assumed constant, and $J_{\text{col}}$ is the collision integral

$$J_{\text{col}} = \int (f' l'_1 - f l哪儿) \cdot (\xi - \xi哪儿) \, ds \, d\Omega,$$

(2)

where $f = f(t, r, \xi), f_1 = F(t, r, \xi_1), f' = f(t, r, \xi'), f'_1 = f(t, r, \xi'_1),$ $\sigma$ is the differential effective collision section, $\xi$ and $\xi_1$ are particle velocities before collision, and $\xi', \xi'_1$ are particle velocities after collision. The unit vector $s$ is directed along the line joining the centers of mass of the colliding particles and $d\Omega = \sin \chi d\chi d\theta$ is a solid angle element.

For an isothermal ($T = \text{const}$) plane-layered atmosphere the local equilibrium state corresponds to a Maxwell–Boltzmann distribution $f_0$, which is an exact solution of kinetic equation (1):

$$f_0 = n_0 \left(\frac{M}{2\pi kT}\right)^{3/2} \exp \left(-\frac{M \xi^2}{2kT} - z/H\right).$$

(3)

The following notation was used in Eq. (3): $n_0$, concentration of particles with mass $M$ at level $z = 0$; $k$, Boltzmann's constant; $H = \kappa T/Mg$, height of the homogeneous atmosphere. In the presence of weak perturbations the distribution function $f(t, r, \xi)$ may be written in the form

$$f = f_0 [1 + \varphi(t, r, \xi)] \quad (\xi \ll 1).$$

(4)

Linearizing Eq. (1) and introducing the new variables

$$\varphi = \xi (M/kT)^{1/2}, \quad \tau = t (kT/M)^{1/2},$$

we have

$$\frac{\partial \varphi}{\partial \tau} + \varphi \frac{\partial \varphi}{\partial r} - \frac{1}{H} \frac{\partial \varphi}{\partial \varphi} = J_\mu = \exp(-\frac{\varphi^2}{2}) \left(\frac{M}{kT}\right)^{1/2} |(\varphi - \varphi_0), \, s | |[\varphi' - \varphi'' - \varphi'_0 + \varphi''] \, ds \, d\Omega. \quad (5)$$

The collision integral of Eq. (5) without the term $\exp(-z/H)$ corresponds to the case of a homogeneous medium. For this case the eigenfunctions $\varphi_{rlm}$ and eigenvalues $\lambda_{rlm}$ of the collision operator $(J_{\text{col}}(\varphi_{rlm}) = \lambda_{rlm} \varphi_{rlm})$ for Maxwellian molecules are defined by well-known relationships [4, 5, 7, 8]. For example, the functions $\varphi_{rlm}$ are expressed in terms of Sonin polynomials $\sin^{l+1/2}$ and Legendre polynomials $P_l^m$ in the form

$$\varphi_{rlm} = \frac{(2l+1)^{1/2}}{(2l+1)^{3/2}} \left(\frac{2l+1}{2l+1}\right)^{1/2} S_{l+1/2} \left(\frac{1}{2} \varphi^2\right) P_l^m (\cos \theta) \, \varphi^l \exp(i\mu\chi),$$

(6)

where $\Gamma(r + l + 3/2)$ is a gamma function, $\cos \theta = \nu_z/\nu, \, \theta$ is the polar angle, and $\chi$ is the azimuthal angle of a spherical coordinate system in velocity space. The functions $\varphi_{rlm}$ are orthogonal with a weight $\Phi(v) = (2\pi)^{-3/2} \exp(-\nu^2/2)$.

We will seek a wave type solution to Eq. (5):

$$\varphi = \varphi(\xi) h(\varphi) \exp[i(k_x x + k_z z - \omega_0 t)],$$

(7)

$$h(\varphi) = \sum_a a_{rlm} \varphi_{rlm} = \sum_{l,m} a_m \varphi_m,$$

where the frequency $\omega_0 = (M/kT)^{1/2}$ and $a_m$ are the coefficients of an expansion of function $h(\varphi)$ in a series in eigenfunctions $\varphi_m$. Introduction of the term $\varphi(\xi)$ considers inhomogeneity of the medium. For $\varphi_m$ in this example we choose several first functions $\varphi_m$ corresponding to the eigenvalues $\lambda_{rlm} = 0$. For two-dimensional perturbations (7) these functions will be $\varphi_1 = \varphi_0 = 1, \, \varphi_2 = \varphi_{x0} = \nu_z, \, \varphi_3 = \varphi_{100} = \sqrt{3/2}[1 - (1/3)\nu^2], \, \varphi_4 = \varphi_{x1} = \nu_x$.

With use of Eq. (7), Eq. (5) takes on the form

$$\sum_{m} a_m \left[ i \left[(\kappa \varphi) - \omega_0 \right] \varphi_m + a \sigma_x \varphi_m - \frac{1}{H} \frac{\partial \varphi_m}{\partial \varphi_x} \right] = 0.$$

(8)

Multiplying Eq. (8) by $\Phi(\nu)\nu^l(\nu)$ and integrating over velocity $\nu$ we obtain

$$\sum_{lm} a_m \left[ -i \omega_0 \delta_{lm} + (a + i k_x) M_{lm} + i k_z B_{lm} - \frac{1}{H} R_{lm} \right] = 0,$$

(9)