SELF-SIMILAR PROBLEMS OF PROPAGATION OF AN EXTENDED HYDROFRACATURE IN A PERMEABLE MEDIUM

Yu. N. Gordeev

The propagation of an extended hydrofracture in a permeable elastic medium under the influence of an injected viscous fluid is considered within the framework of the model proposed in [1, 2]. It is assumed that the motion of the fluid in the fracture is turbulent. The flow of the fluid in the porous medium is described by the filtration equation. In the quasisteady approximation and for locally one-dimensional leakage [3] new self-similarity solutions of the problem of the hydraulic fracture of a permeable reservoir with an exponential self-similar variable are obtained for plane and axial symmetry. The solution of this two-dimensional evolution problem is reduced to the integration of a one-dimensional integral equation. The asymptotic behavior of the solution near the well and the tip of the fracture is analyzed. The difficulties of using the quasisteady approximation for solving problems of the hydraulic fracture of permeable reservoirs are discussed. Other similarity solutions of the problem of the propagation of plane hydrofractures in the locally one-dimensional leakage approximation were considered in [3, 4] and for leakage constant along the surface of the fracture in [5—7].

1. FORMULATION OF THE PROBLEM

We will consider two formulations of the problem of the hydraulic fracture of a permeable medium: for plane and axisymmetric fractures in a uniform compressive stress field.

Plane problem. At the boundary of a fluid-saturated porous permeable half-space at the initial instant of time (t = 0) the fluid pressure increases abruptly from the pore pressure $p_\infty$ to the pressure $P_0$, which is then kept constant. Under the influence of the wedge effect of the flow of fluid into the porous half-space a plane fracture begins to propagate from the boundary. In this case fluid percolates into the reservoir both across the boundary of the half-space and across the edges of the fracture (Fig. 1a).

Axisymmetric problem. At the initial instant of time let fluid begin to be injected into a well with the flow rate $q$. An axisymmetric hydrofracture propagates into the fluid-saturated permeable reservoir with pore pressure $p_\infty$. Fracture fluid percolates into the reservoir through the walls of the well and the edges of the crack (Fig. 1b). We will consider the case of turbulent motion of the fracture fluid in the fracture. For describing this flow we will use a quadratic resistance law and the continuity equation

$$\frac{\partial w}{\partial t} + \frac{1}{r^n} \frac{\partial}{\partial r} \left( r^n u w \right) = -2u_L$$

(1.1)

$$\frac{\partial}{\partial r} p = -\lambda_s \frac{p_0 u}{w}$$

(1.2)

Here, $p$ is the fracture fluid pressure, $\rho_0 = \text{const}$ is the fluid density, $u$ is the velocity of the fracture fluid in the fracture, $u_L$ is the local velocity of the fluid leaking through the fracture surfaces, $w$ is the fracture opening, $n$ is the symmetry index of the problem ($n = 0$ for a plane and $n = 1$ for an axisymmetric fracture), and $\lambda_s$ is the flow resistance coefficient (in general this depends on the Reynolds number Re of the flow and the roughness of the edges of the fracture; at high values of Re $\lambda_s$ is constant).

It is assumed that the filtration of the fluid in the porous medium can be described by the piezoconductivity equation ($\kappa$ is the piezoconductivity coefficient)

$$\frac{\partial}{\partial t} p_1 = \kappa \left[ \frac{\partial^2}{\partial y^2} p_1 + \frac{1}{r^n} \frac{\partial}{\partial r} \left( r^n \frac{\partial}{\partial r} p_1 \right) \right]$$

(1.3)

where $p_1(r, y, t)$ is the fluid pressure in the reservoir, and $y$ is the coordinate in a direction perpendicular to the plane of the
The length of the fracture can be determined from the fracture criterion [8]

\[ \int dr r^n \left[ p(r, t) - p_h \right] \left( r^2 - r_0^2 \right)^{\alpha} = \frac{K_1}{\sqrt{2l(t)}} \]  

(1.4)

\( p_h \) is the rock pressure at the depth \( h \); \( K_1 \) is the cohesion modulus of the rock, \( l(t) \) is the length of the fracture at time \( t \), and \( r_0 = 0 \) for \( n = 0 \) and \( r_0 > 0 \) when \( n > 0 \) is the radius of the well.

We will consider the propagation of the fracture in the quasisteady approximation; accordingly, the fracture opening is given by Sneddon’s well-known solution [9], into which time enters as a parameter:

\[ w(r, t) = \frac{4(1-v)l(t)}{\pi G} \int_0^l \frac{[p(\xi, t) - p_0] \xi^2 \theta^{1-\alpha} d\theta}{\sqrt{\theta^2 - \epsilon^2}} \]

\[ \eta = \frac{r}{l(t)}, \quad g(t) = \frac{r_0}{l(t)} \]  

(1.5)

\( G \) is the shear modulus, and \( v \) is Poisson’s ratio.

The system of equations (1.1)–(1.5) was investigated for the initial and boundary conditions:

\[ p_0(r, y, t=0) = p_0 \]  

(1.6)

\[ p_0(r, y=0, t) = p(r, t), \quad u(r, t) = -\frac{\mu}{k} \frac{\partial}{\partial y} p_1(r, y=0, t) \]  

(1.7)

\[ p(r=0, t) = p_0(r=0, y, t), \]  

(1.8)

\[ q(t) = 2\pi \lim_{r \to r_0} \left\{ -\frac{k}{\mu} \int_{r_n}^{r_0} dy \frac{\partial p}{\partial y} + wu \right\} \]  

(1.9)

where \( h_2 \) is the distance to the surface of the earth, and \( h_1 \) is the distance to the bottom of the well.

2. SELF-SIMILAR PROBLEM

In [10] it was shown that in soil mechanics for any significant fracture length the cohesion modulus in expression (1.4) can be neglected, i.e., we may assume that \( K_1(2l(t)-2n)^{1/2} \approx 0 \).

We introduce the following dimensionless variables and parameters:

\[ \rho(r, t) = p_h P(\xi, \tau), \quad p_0(r, y, t) = p_0 P(\xi, \eta, \tau), \quad \xi = \frac{r}{l_f(\tau)}, \quad \eta = \frac{y}{y_0}, \quad t = t_s \tau, \quad \epsilon = \frac{l_0}{l_s} u_0 \]

\[ l(t) = l_f(\tau), \quad f(\tau) = 1, \quad w(r, t) = w_0 f(\tau) W(\xi, \tau), \quad u(r, t) = u_s U(\xi, \tau), \quad u_s(t) = u_s V(\xi, \tau), \quad \frac{u_s}{u_0} = \frac{1}{2} \frac{w_0}{l_0} u_0 \]

\[ u_s = \left( \frac{1}{\lambda_s} \frac{w_0 p_0}{l_0 p_0} \right)^{\lambda_s}, \quad y_0 = \frac{k p_0}{\mu u_s}, \]  

\[ \tau_s = \frac{y_0^2}{\lambda_s}, \quad w_0 = \frac{4(1-v)}{\pi} \frac{h}{G} l_0, \quad N = \frac{p_0}{p_h}, \quad M = \frac{\rho_0}{\rho_h} \]

\[ q = q_s f(\tau) Q(\xi, \tau), \quad q_s = (2\pi)^{n} u_s l_0 \]

\[ \beta(\tau) = \frac{y_0^2}{(l_f(\tau))^2} \]