THE STATISTICAL CHARACTERISTICS OF ADAPTIVE ANTENNA ARRAYS WHEN PROCESSING DISCRETE SIGNALS WITH CORRELATED READOUTS

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The most important statistical characteristics of adaptive antenna arrays are determined. These are the mean value and covariance matrix of the vector of the weighting factors, and the powers of the fluctuations of the weighting factors and of the output signal of the array. It is found that under steady-state operating conditions the fluctuations of the vector of the weighting factors lead to a bias of its mean value for an adaptive antenna array without constraints, but the mean value of the weighting vector of an array with constraints and of an array operating using the criterion of the minimum root mean square error is not biased. It is shown that the effect of fluctuations of the weighting vector on the output signal of an adaptive antenna array may be different and is determined by the value of the correlation coefficient between the input-signal readouts.

1. Introduction

The problem of a statistical analysis of adaptive antenna arrays taking into account the fluctuations in the adjustable weighting factors in the case of continuous time was considered in detail in [1, 2]. In particular, it was shown in [1] that a correct analysis of adaptive systems involves taking into account the non-Gaussian statistical dependence of the vector of the weighting factors and the vector of the input signals. On the basis of these conclusions it was found in [2], for example, that fluctuations in the weighting factors lead to a reduction in the overall power at the output of the array with continuous graded adjustment algorithms. In the case of discrete time, which was considered in [3] for independent readouts of the received signal, the overall output power due to fluctuations in the weighting vector, on the contrary, increases. The condition for the readouts to be independent in practice in the case of digital signal processing is only rarely rigorously satisfied. It is obvious that the case of correlated readouts is "intermediate" between the case studied in [1, 2] and that studied in [3] and is most interesting for practical applications. However, the problem of the effect of fluctuations of the weighting factors on the characteristics of adaptive antenna arrays when there is correlation between the readouts of the received signal has not been investigated in any publications known to us.

In this paper we determine the main single-moment statistical characteristics of a narrow-band adaptive antenna array when processing signals with correlated readouts. The method used enables us to analyze the statistical characteristics of different adaptive antenna arrays (with constraints on the radiation pattern, and also operating using the criteria of minimum root mean square error and maximum signal/noise ratio).

2. Formulation of the Problem

Consider the operation of an N-element narrow-band adaptive antenna array with constraints, a block diagram of which is shown in Fig. 1. Multiple linear constraints on the spatial characteristics of the adaptive antenna array in this scheme are introduced using a matrix filter in the control circuit [4, 5]

\[ P = I - C(\mathbf{C}^+\mathbf{C})^{-1}\mathbf{C}^+, \]

where $\mathbf{I}$ is the $N \times N$ unit matrix, and $\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_L]$ is an $N \times L$ matrix of constraints, the columns of which are linearly independent constraint vectors $\mathbf{C}_i$ ($L$ is the number of constraints introduced). As was shown in [4, 5], the matrix operator $\mathbf{P}$ projects estimates of the gradient of the output power of the array in the space of the weighting factors $\mathbf{W} = \{W_1, W_2, \ldots, W_N\}^T$ onto the space (hyperplane) of the constraints

$$\mathbf{C}^+ \mathbf{W} = \mathbf{H},$$

where $\mathbf{H} = \{H_1, H_2, \ldots, H_L\}^T$ is an $L$-dimensional vector whose components specify the fixed gains of the adaptive antenna array in the directions of the vectors $\mathbf{C}_i^*$.

The stochastic differential equation describing the behavior of the weighting factors of the adaptive antenna array with constraints in discrete time will have the form [5]

$$\mathbf{W}(k + 1) = \mathbf{P} \left\{ \mathbf{W}(k) - \mu \mathbf{X}^*(k) \mathbf{X}^T(k) \mathbf{W}(k) \right\} + \mathbf{W}_q,$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_N\}^T$ is a vector of the complex envelopes of the input signals at the receiving elements of the array, $\mathbf{W}_q = \mathbf{C}(\mathbf{C}^+\mathbf{C})^{-1}\mathbf{H}$ is the vector of the complex weighting factors corresponding to the "desired" quiescent radiation pattern (that is, when there is no external interference) [5], and $\mu$ is the gain in the feedback circuit (the adaptation factor, governing the rate of convergence).

Note that Eq. (2) for an adaptive antenna array with constraints is fairly general. One can obtain from it, as special cases, the equations for adaptive antenna arrays operating using other optimality criteria. Thus, assuming

$$\mathbf{P} = \mathbf{I}, \quad \mathbf{W}_q = \mu \mu_0 \mathbf{S}^*,$$

we obtain from (2) the equation for the weighting vector of an adaptive antenna array having maximum signal/noise ratio [6, 7]

$$\mathbf{W}(k + 1) = \mathbf{W}(k) + \mu \left\{ \mu_0 \mathbf{S}^* - \mathbf{X}^*(k) \mathbf{X}^T(k) \mathbf{W}(k) \right\},$$

where $\mathbf{S}$ is the vector-phaser of the useful signal and $\mu_0$ is an arbitrary constant having the dimensions of power.

The equation of a graded algorithm of an adaptive antenna array with minimum root mean square error [3]

$$\mathbf{W}(k + 1) = \mathbf{W}(k) + \mu \mathbf{X}^*(k) \left\{ d(k) - \mathbf{X}^T(k) \mathbf{W}(k) \right\}$$