DETECTION OF NOISE SIGNALS IN MULTICHANNEL RECEPTION UNDER THE CONDITIONS OF PARAMETRIC A PRIORI UNCERTAINTY

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The method of maximum likelihood is used to construct tests with a fixed probability of a false alarm, the powers of which do not depend on the unknown intensities of the signal and interference and the position of the signal but are determined only by the signal-to-noise ratio. The asymptotic statistical distribution in the absence of a signal is found, on which a detector of a weak noise signal is based. The constructed detectors are compared with known detectors by means of statistical simulation on a computer.

Introduction

The theory of signal detection is a well-known and well-developed problem of statistical radio engineering. Research on special problems of detection theory that are closer to reality and free from some of the restrictions usually adopted are continuing, however. One of them is the problem of the statistical synthesis of detection algorithms under the conditions of parametric a priori uncertainty with respect to the signal and interference, rejecting the use of a priori distributions of the unknown parameters and with no training examples from realizations of observable processes. In the present paper we discuss the solution of one class of problems of this type. Optimization of detection under such conditions is usually achieved by seeking the uniformly most powerful (UMP) criteria. Unfortunately, UMP criteria do not exist in many of the most realistic problems; a number of complicated problems of the optimization of detection can be solved on the basis of the principles of similarity and nonbias, however, while if a certain symmetry is present in the initial data, they can be solved using the invariance principle [1]. For the class of problems being considered here, the principles of nonbias and invariance do not lead to optimal solutions [2]. An alternative approach to obtaining optimal solutions is the minimax approach, but its use in specific problems leads to difficulties and can be successful mainly in cases when UMP invariant criteria exist [2]. The problems being considered here are solved by the method of maximum likelihood (ML), which is attractive because a test (criterion) constructed by the ML method is asymptotically more accurate [3] than any other valid test of the same hypothesis, under certain conditions (which are satisfied, assuming that the signal and interference are Gaussian). In our case, the ML method leads to the construction of criteria that are fairly simple to implement and possess the property of similarity, which guarantees a fixed probability of a false alarm, and their power does not depend on unknown parameters and is determined only by the signal-to-noise ratio.

Below we consider the solution to the problem of the detection of various noise signals having a given nature of intensity variation against an isotropic stationary background for multichannel (with respect to the angular coordinates) reception at one point. We assume that the set of observations used for the statistical analysis of the radar environment was obtained at one point of space using directional antennas. The specific nature of the signal intensity variation is determined by the antenna’s beam pattern or the velocity of the source. The signal and interference are assumed to be Gaussian white noises with unknown intensities. The numbers of the channels in which the signal may appear are unknown.

Such problems are typical in radio-astronomical observations or the tracking of radio sources in space.

1. Problems under Consideration

We assume that M independent, repeated samples
from a realization taken over an observation time equal to one survey are to be analyzed. The samples are from distributions \( N(0; \sigma_i^2) \), \( i = 1, M \), where \( M \) is the number of channels.

The hypothesis that a signal is absent is tested:

\[
H: \sigma_i^2 = \sigma_0^2, \quad i = 1, M, \quad \sigma_0^2 > 0 \quad \text{where} \quad \sigma_0^2 \quad \text{is the unknown intensity of the noise background, against one of the following alternatives:}
\]

\[
H_1: \quad \sigma_i^2 > \sigma_0^2, \quad \sigma_i^2 = \sigma_0^2, \quad i \neq l, \quad i = 1, M,
\]

\( \sigma_i^2 = \sigma_0^2 + \sigma_s^2, \quad \sigma_s^2 > 0, \quad l = 1, M \), where \( \sigma_s^2 \) is the unknown signal intensity, means that a stationary noise signal is present in the \( l \)th channel;

\[
H_2: \quad \sigma_{i+l}^2 = \ldots = \sigma_{i+L}^2 > \sigma_0^2, \quad \sigma_l^2 = \sigma_0^2,
\]

\( i = 1, \ldots, i, i + L + 1, \ldots, M \), i.e., \( i \not\in \{l + 1, l + L\} \), \( l = 0, M - L \), means that a stationary noise signal is present in \( L \) channels (from \( l + 1 \) to \( l + L \)), with \( L = \frac{M}{2}, M - 1 \) being given;

\[
H_3: \quad \sigma_{i+l}^2 = \alpha_i \sigma_0^2, \quad i = 1, L, \quad l = 0, M - L,
\]

\( \sigma_k^2 = \sigma_0^2, \quad k \not\in \{l + 1, l + L\} \), where \( L = \frac{M}{2}, M - 1 \) is given and \( \alpha_i > 1 \) are known, means that a piecewise-stationary noise signal (here and below, the signal is stationary in each channel) of known length (the signal occupies \( L \) channels) is present; the character of the nonstationarity is determined by the known signal-to-noise ratio \( \lambda_i = (\alpha_i - 1)^{1/2} \) in each of these channels; \( K \): any uncertainty (inequality) of dispersions \( \sigma_i^2, \quad i = 1, M \), means that one or several piecewise-stationary noise signals of arbitrary length and shape are present (by the shape of a noise signal we mean the character of the variation of its intensity);

\[
H_{4n}: \quad \sigma_{i+L}^2 = \sigma_0^2 + g_i \sigma_s^2 / n, \quad i = 1, L, \quad l = 0, M - L, \quad g_i \leq 1,
\]

\( \sigma_k^2 = \sigma_0^2, \quad k \not\in \{l + 1, l + L\} \), where \( L = \frac{M}{2}, M - 1 \) is given and \( g_i \) are known, means that a weak, piecewise-stationary noise signal of known length \( L \) and shape \( g_i \) is present. The signal's intensity \( g_i \sigma_s^2 / n \) decreases with increasing \( n \) as \( 1/n \). Here \( H_{4n} \) is a sequence of converging alternatives (the rate of convergence with the hypothesis \( H \) is \( 1/n \)).

The alternatives described above were introduced into the analysis to make fuller allowance for the available information about the signals under consideration in order to increase the power of their detection in comparison with nonparametric and ML detection, which do not employ such particular knowledge about the processes being observed.

The formulation of each alternative (except for \( K \)) incorporates information about the character of the intensity variation of the noise signal being detected, which is available, as a rule, in actual problems of the detection of noise signals against an isotropic, stationary noise background in multichannel reception. Information is incorporated into \( H_1 \) and \( H_2 \) about the constancy of the signal's intensity (the case of a stationary signal) and the possibility that it is present in only one \( H_1 \) or in \( L \) successive channels \( H_2 \). \( H_3 \) uses the greatest amount of \textit{a priori} information and describes the presence of a nonstationary signal in \( L \) successive channels, the intensity of which varies stepwise with channel number, with the character of the intensity variation being given by the known signal-to-noise ratio in each channel. \( H_{4n} \) differs from \( H_3 \) in that to describe the character of the intensity variation it uses not the known signal-to-noise ratio but only its law of variation from channel to channel, given by the set of coefficients \( g_i, \quad i = 1, L \), besides which the signal intensity decreases at a given rate as the volume of observations increases, which corresponds to the case of a departing source. The alternative \( K \) has the most general form and includes all the situations that differ from the hypothesis \( H \). Its use is appropriate when any \textit{a priori} information about the character of the signal's intensity variation is absent. Using the alternative \( K \) when there is any \textit{a priori} information about the signal leads to a lower detection power than when that information is taken into account in formulating the alternative.

2. Known Uniformity Criteria

A UMP unbiased criterion for testing the hypothesis \( H \) exists only in the case of \( M = 2 \) \cite{2}. In this problem there is no group for which a UMP invariant criterion exists \cite{2}. Criteria for the hypothesis \( H \) for \( M > 2 \) are known that do not,