It is known [8] that for sufficiently large \( \lambda \) a.s.

\[
\lim_{n \to \infty} \sum_{i=1}^{n} a_i \varepsilon_i (B_n^2 \ln \ln B_n^2)^{\gamma/2} > 0,
\]

(19)

\[
B_n^2 = \sum_{i=1}^{n} a_i^2 \sim (2\lambda)^{-1} \exp (2\lambda n / \ln n),
\]

\[
B_n^2 \ln \ln B_n^2 \sim (2\lambda)^{-1} \exp (2\lambda n / \ln n) \ln n \sim \frac{\lambda}{2} A_n^2.
\]

(20)

Relations (19), (20) contradict Eq. (1); thereby Theorem 3 is proved.

Remark. Mourier [9], using the strong law of large numbers in \( \mathbb{R}^3 \), proved that a similar assertion holds also in a separable Banach space. The analysis of this proof shows that it is transferred without essential changes to weighted Riesz sums. Therefore, Proposition 1 and Theorems 1-3 will be valid for sequences of independent identically distributed random elements in separable Banach spaces.

LITERATURE CITED

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POSITIVE SEMICHARACTERS AND THE LAPLACE TRANSFORM

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It is shown that the image of the conical semigroup \( S \) under the natural mapping of it to its second semigroup of positive semicharacters \( \hat{S}_+ \) is a total subset of the space of continuous functions on \( \hat{S}_+ \) approaching zero at infinity. This allows one to establish a number of properties of the semigroup \( \hat{S}_+ \) and of the Laplace transform of measures on it.

In the treatise [1] (Chap. IX, Sec. 5) there is considered, in particular, the Laplace transform of measures on a semigroup \( M \) under certain conditions on the semigroup of its semicharacters. It follows from Theorem 1 of this paper that these conditions are satisfied if \( M \) is a semigroup of nonnegative semicharacters of a conical semigroup (see the definitions below). The properties (somewhat modified) stemming from this fact due to the results from [1] of the Laplace transform allow one to more deeply study the semigroup of nonnegative semicharacters and obtain the internal characteristic of the Laplace transform of a complex meas-
Everywhere below $G$ is a locally compact abelian group with an identity $e$, $S$ is an open subsemigroup of $G$, for which $e$ is a point of tangency (in [4] such semigroups are called conical). By a semicharacter of the semigroup $S$ we will mean a nontrivial continuous homomorphism of this semigroup to the unit disk with the operation of multiplication of complex numbers. By $\widehat{S}$ let us denote the set of all semicharacters of $S$, equipped with the topology of uniform convergence on compact subsets of $S$, and by $S_+$ the subspace of this space consisting of nonnegative semicharacters. The space $\widehat{S}_+$ is locally compact as a closed subset of the space $\widehat{S}$ of maximal ideals of the algebra $L^1(S)$ [5]. Furthermore, $\widehat{S}_+$ is a semigroup with respect to pointwise multiplication. This follows from Lemma 1 of this paper. For $\rho \in \widehat{S}_+$, let $S(\rho) = \{ t \in S : \rho(t) > 0 \}$, and let $G(\rho)$ be the subgroup of $G$ generated by $S(\rho)$. Then $G(\rho)$ is open-closed in $G$, and $S(\rho)$ is an open-closed subsemigroup of the semigroup $S$, since $S(\rho) = G(\rho) \cap S$. The complement $N(\rho) = S \setminus S(\rho)$ is an open-closed ideal of $S$. Let $S_1 = S \cup \{e\}$. Finally, let $\hat{e}(\rho) = \rho(t)$ for $t \in \widehat{S}$, $\hat{e}(\rho) = 1$.

1. The Basic Theorem. At the base of the subsequent considerations lies the following result.

**THEOREM 1.** The family $\{ \hat{f} : \hat{f} \in \widehat{S} \}$ is total in the space $C_0(\widehat{S}_+)$ of continuous functions on $\widehat{S}_+$ approaching zero at infinity.

Let us preface the proof of Theorem 1 by two lemmas.

**LEMMA 1.** Each semicharacter $\rho \in \widehat{S}_+$ is continued in a unique manner to a continuous homomorphism $\rho$ of the group $G(\rho)$ to the multiplicative semigroup $\mathbb{R}^+$ of nonnegative real numbers.

**Proof.** For $s, t \in S(\rho)$ let $\rho(st^{-1}) = \rho(s)/\rho(t)$. The correctness of this definition is easily verified. It is also evident that $\overline{\rho}$ is a homomorphism. Let us prove its continuity. Let $x_0 \in G(\rho)$. There is $t_0 \in S(\rho)$, such that $s_0 = t_0 x_0 \in S(\rho)$. For $\varepsilon > 0$ there is a neighborhood $U_0$ of the point $s_0$, such that $|\rho(s) - \rho(s_0)| < \varepsilon \rho(t_0)$ for $s \in U_0$. Let us consider $U = t_0^{-1} U_0$. This is a neighborhood of the point $x_0$ and for $x \in U$ ($x = t_0^{-1} s$, where $s \in U_0$) we have

$$|\rho(x) - \rho(x_0)| = |\rho(s) - \rho(s_0)|/\rho(t_0) < \varepsilon,$$

which in fact completes the proof of the lemma.

**LEMMA 2.** For any $\Lambda \subset \widehat{S}_+$ the set $\Lambda \cap \{ S(\rho) : \rho \in \Lambda \}$ is open and in the case of nonemptiness contains the intersection $\Lambda \cap S$ for some neighborhood $U$ of the identity of the group $G$.

**Proof.** Let us assume that $t \in \Lambda \cap \{ S(\rho) : \rho \in \Lambda \}$. Then $\Lambda = S \setminus \Lambda \cap \{ S(\rho) : \rho \in \Lambda \} = \bigcup \{ S(\rho) : \rho \in \Lambda \} = \Lambda \cup N(\rho)$. If one assumes that $e$ is a point of tangency for $N$, then $S^{-1} \cap N = \emptyset$. Therefore, there are $s \in S$, $a \in N$, such that $t = as \notin N$. Consequently, there exists a neighborhood $U$ of the point $e$ such that $U = \emptyset$. Therefore, $U \cap S$ is a nonempty open subset of $\Lambda \cap \{ S(\rho) : \rho \in \Lambda \}$. Let then the subgroup $H = \bigcap \{ G(\rho) : \rho \in \Lambda \}$ of the group $G$ is open. It remains to observe that $\bigcap \{ S(\rho) : \rho \in \Lambda \} = H \cap S$.

**Proof of Theorem 1.** Let us observe that the functions $\hat{f}$ are continuous on $\widehat{S}_+$. If the space $\widehat{S}_+$ is compact, then, evidently, the Weierstrass-Stone Theorem is applicable to the family $\{ \hat{f} : \hat{f} \in \widehat{S} \} \subset C(\widehat{S}_+) = C_0(\widehat{S}_+)$. We will further assume that $\widehat{S}_+$ is not compact. Let us fix $t \in S$ and show that $\hat{t} \in C_0(\widehat{S}_+)$. Let the direct $(p_x : \alpha \in A) \subset \widehat{S}_+$ approach infinity in $\widehat{S}_+$. Let $A' = \{ \alpha : \exists \alpha \in A \}$. Let us consider the following two cases.

1. There exists $\alpha_0 \in A$, such that $\alpha \leq \alpha_0$, for all $\alpha \in A'$. Then $\lim_{\alpha \in A} \hat{f}(p_\alpha) = 0$.

2. Such an $\alpha_0$ does not exist. This means that $(p_\alpha : \alpha \in A')$ is a subdirection of the initial direction. Let us show that in this case $\lim_{\alpha \in A} \hat{f}(p_\alpha) = 0$. Let $H = \bigcap \{ G(\rho) : \alpha \in A' \}$. By Lemma 2 $H$ is an open subgroup of $G$. As is known [6], $H$ contains an open subgroup of the form $\mathbb{R}^n \times F$, where $n \in \mathbb{Z}_+$, and the group $F$ is compact.